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Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

Stable Matching Problem

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- n candidates and n jobs.

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- n candidates and n jobs.
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- Each candidate has a ranked preference list of jobs.

How should they be matched?

The best laid plans..

Consider the pairs..

- (Anthony) Davis and Pelicans
- (Lonzo) Ball and Lakers

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Davis prefers the Lakers.

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Uh..oh.

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Uh..oh. Sad Lonzo and Pelicans.

So..

Produce a matching where there are no crazy moves!

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Definition: A **matching** is disjoint set of n job-candidate pairs.

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Example: A matching $S = \{(\mathit{Lakers}, \mathit{Ball}); (\mathit{Pelicans}, \mathit{Davis})\}$.

So..

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Definition: A **rogue couple** b, g^* for a pairing S:
b and g^* prefer each other to their partners in S

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Definition: A **rogue couple** b, g^* for a pairing S:
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Example: Davis and Lakers are a rogue couple in S.

Example.

Jobs			
A	1	2	3
B	1	2	3
C	2	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

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	Day 1	Day 2	Day 3	Day 4	Day 5
1					
2					
3					

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3					

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The Propose and Reject Algorithm.

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Each Day:

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Terminates in $\leq n^2$ steps!

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Improvement Lemma: It just gets better for candidates

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If on day t a candidate g has a job b on a string,

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Example: Candidate "Alice" has job "Amalgamated Concrete" on string on day 5.

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Example: Candidate "Alice" has job "Amalgamated Concrete" on string on day 5.

She has job "Amalgamated Asphalt" on string on day 7.

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Poll

Question: It just gets better for candidates, because?

(A) Induction on days.

(B) When the economy is good.

(C) The candidate can always keep the job on the string.

Matching when done.

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Question: The argument for termination uses.

(A) Implies: no unmatched job at end.

(B) Improvement Lemma: every candidate matched.

(C) Algorithm: unmatched job would ask everyone.

(D) Implies: every one gets their favorite job.

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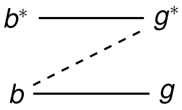

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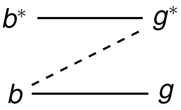

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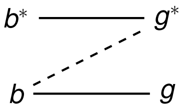

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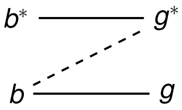

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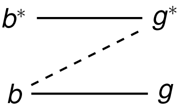

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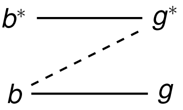

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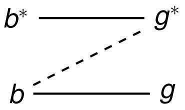

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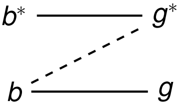

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Is the Job-Proposes better for jobs?

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Question: The SMA produces a stable pairing is a proof by?

(A) Contradiction.

(B) Uses the improment lemma.

(C) Induction.

(D) Direct.

Understanding Optimality: by example.

A:	1,2	1:	A,B
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Pairing T: $(A,2),(B,1)$.

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Which is optimal for A?

Understanding Optimality: by example.

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Which is optimal for 1?

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Job Propose and Candidate Reject is optimal!

For jobs?

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For jobs? For candidates?

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Theorem: Job Propose and Reject produces a job-optimal pairing.

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Proof:

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Assume not:

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Assume not: there is a job b does not get optimal candidate, g.

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b^* - knocks b off of g's string on day t

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b^* - knocks b off of g's string on day $t \implies g$ prefers b^* to b

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By choice of t, b^* likes g at least as much as optimal candidate.

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Rogue couple for S.

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So S is not a stable pairing.

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How does one make it better for candidates?

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Propose and Reject - stable matching algorithm. One side proposes.

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