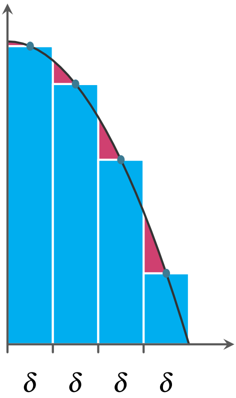
Survey

Fill it out!!

tinyurl.com/cs70-survey

Calculus



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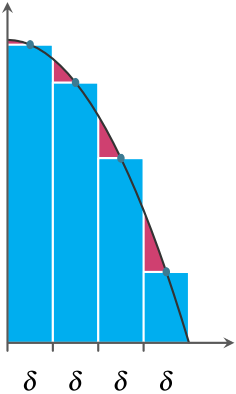

Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

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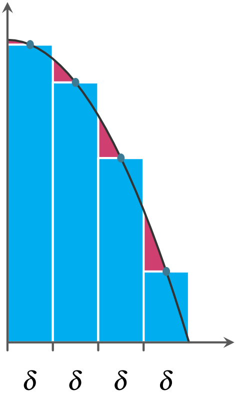

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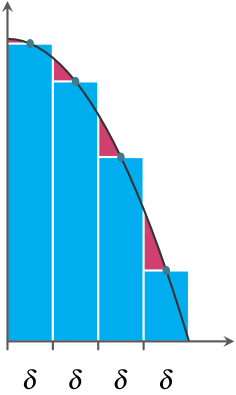

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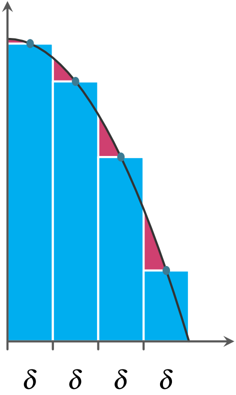

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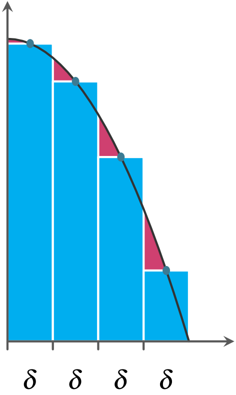

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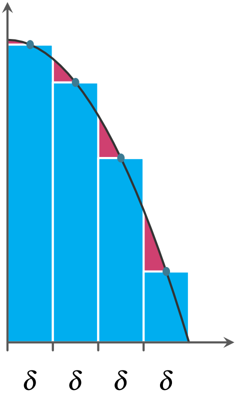

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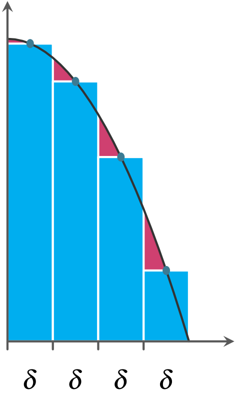

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Thus $F'(x) = f(x)$.

CS70: Continuous Probability.

$\boxed{\text{Continuous Probability 1}}$

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1. Examples
2. Events
3. Continuous Random Variables

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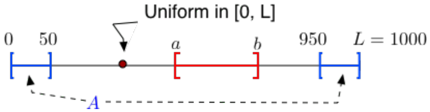

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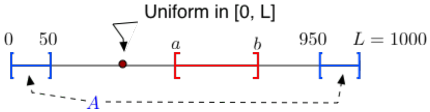

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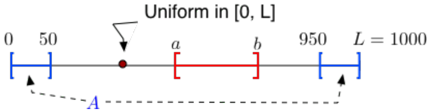

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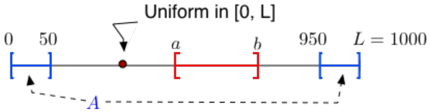

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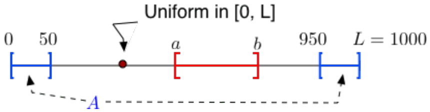

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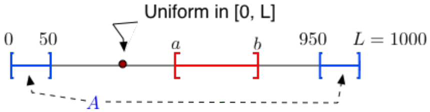

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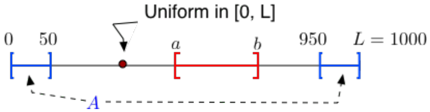

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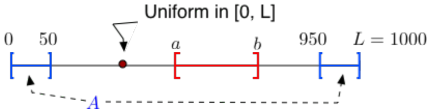

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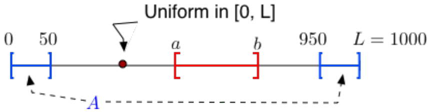

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Makes sense: $b - a$ is the fraction of $[0,1]$ that $[a,b]$ covers.

Poll

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(D) $f(x) = F'_X(x)$.
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Next lecture.

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More generally, events in this space are unions of intervals.

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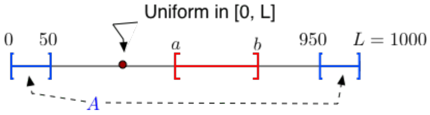

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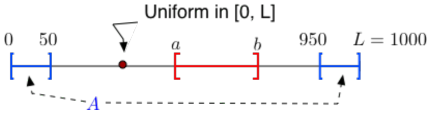
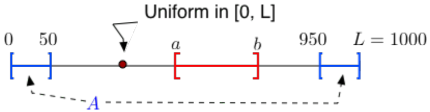


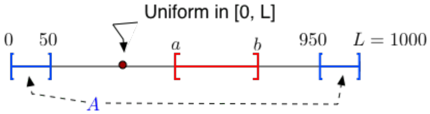
Note: A **radical** change in approach.

Uniformly at Random in $[0, 1]$.



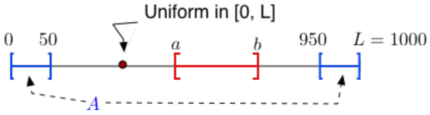
Note: A **radical** change in approach.
Finite prob. space:

Uniformly at Random in $[0,1]$.



Note: A **radical** change in approach.
Finite prob. space: $\Omega = \{1,2,\ldots,N\}$,

Uniformly at Random in $[0,1]$.



Note: A **radical** change in approach.
Finite prob. space: $\Omega = \{1, 2, \ldots, N\}$, with $Pr[\omega] = p_\omega$.

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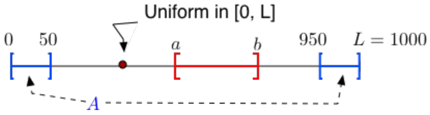

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Finite prob. space: $\Omega = \{1,2,\ldots,N\}$, with $Pr[\omega] = p_\omega$.

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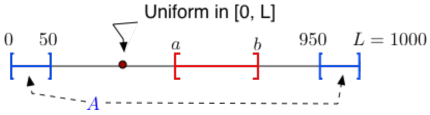
Uniformly at Random in $[0,1]$.



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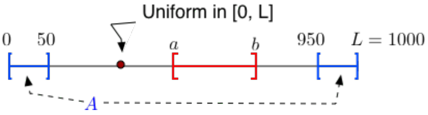
Continuous space:

Uniformly at Random in $[0,1]$.



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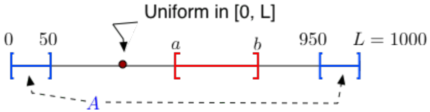

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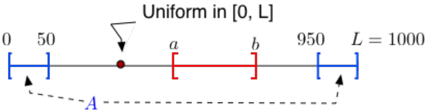
Continuous space: e.g., $\Omega = [0,1]$,
$Pr[\omega]$ is typically 0.

Uniformly at Random in $[0,1]$.



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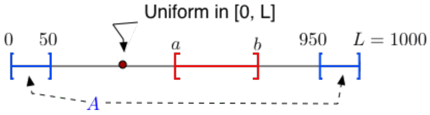
Instead, start with $Pr[A]$ for some events A.

Uniformly at Random in $[0,1]$.



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Continuous space: e.g., $\Omega = [0,1]$,
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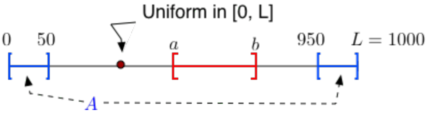
Instead, start with $Pr[A]$ for some events A.
Event A = interval,

Uniformly at Random in $[0, 1]$.



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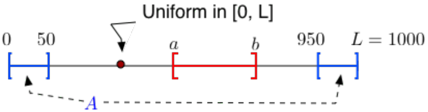
Uniformly at Random in $[0,1]$.



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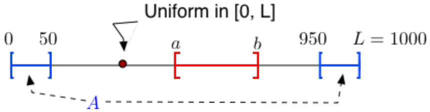
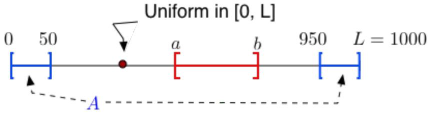
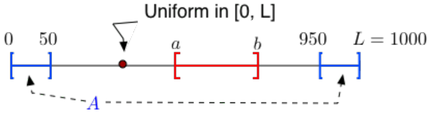


Uniformly at Random in $[0, 1]$.



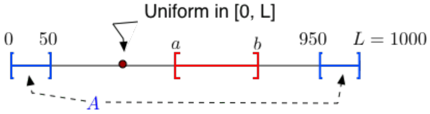
$Pr[X \leq x] = x$ for $x \in [0, 1]$.

Uniformly at Random in $[0, 1]$.



$Pr[X \leq x] = x$ for $x \in [0, 1]$. Also, $Pr[X \leq x] = 0$ for $x < 0$.

Uniformly at Random in $[0,1]$.



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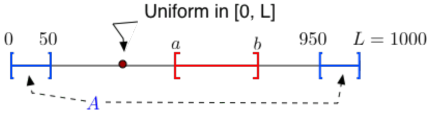
Uniformly at Random in $[0,1]$.



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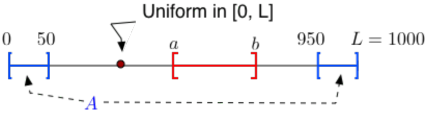
Define $F(x) = Pr[X \leq x]$.

Uniformly at Random in $[0,1]$.



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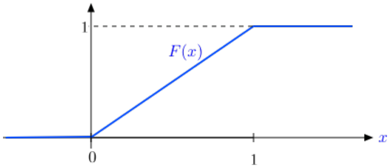

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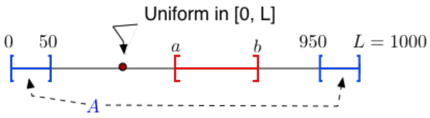

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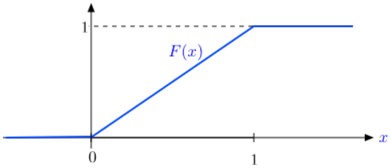
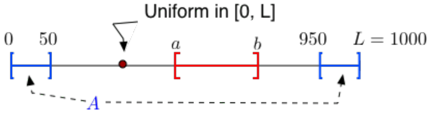


Then we have $Pr[X \in (a, b]] = Pr[X \leq b] - Pr[X \leq a]$

Uniformly at Random in $[0,1]$.



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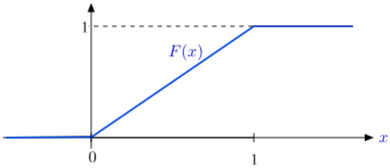

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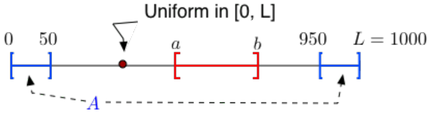

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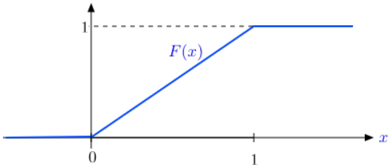
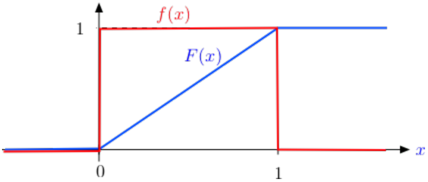


Then we have $Pr[X \in (a, b]] = Pr[X \leq b] - Pr[X \leq a] = F(b) - F(a)$.

Thus, $F(\cdot)$ specifies the probability of all the events!

Uniformly at Random in $[0, 1]$.



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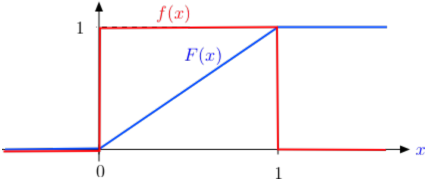

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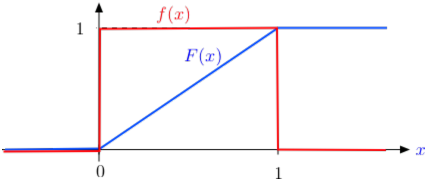
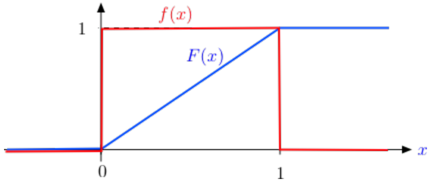


$$Pr[X \in (a, b]] = Pr[X \leq b] - Pr[X \leq a] = F(b) - F(a).$$

An alternative view is to define $f(x) = \frac{d}{dx}F(x) =$

Uniformly at Random in $[0,1]$.



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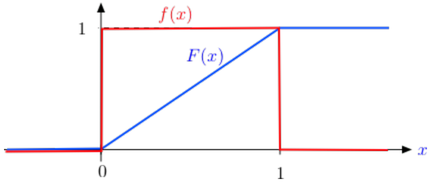
Uniformly at Random in $[0, 1]$.



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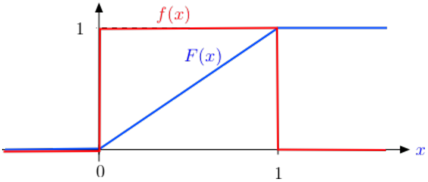

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Thus, the probability of an event is the integral of $f(x)$ over the event:

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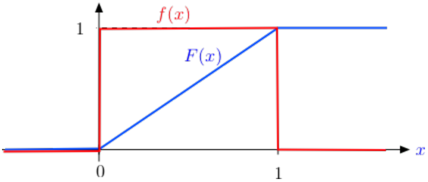

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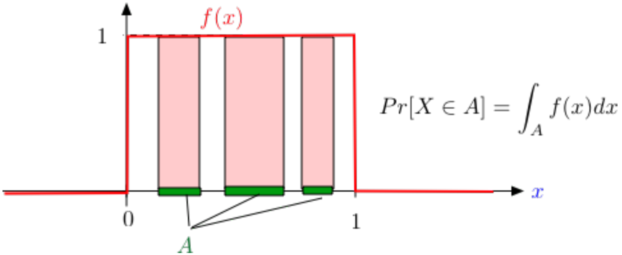
Thus, the probability of an event is the integral of $f(x)$ over the event:

$$Pr[X \in A] = \int_A f(x)dx.$$

Uniformly at Random in $[0, 1]$.



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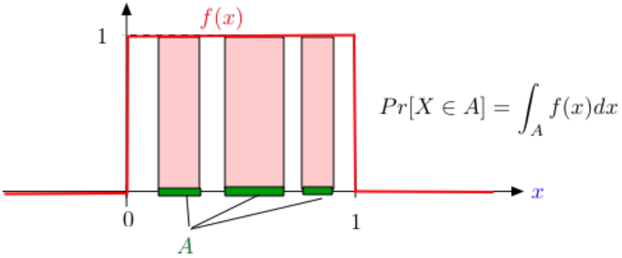
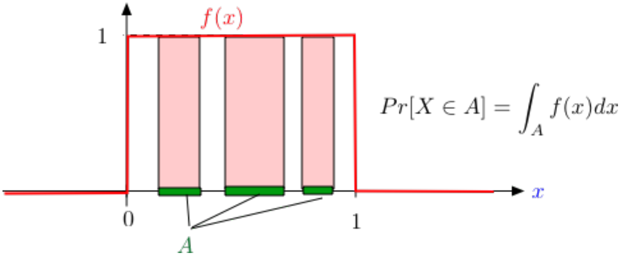


Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$:

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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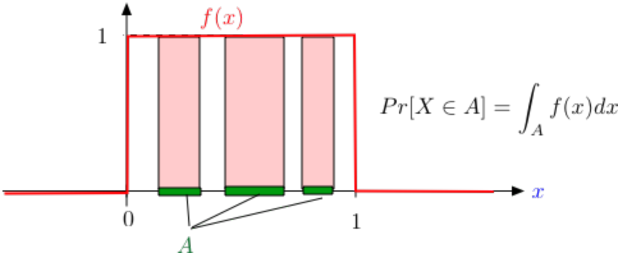
Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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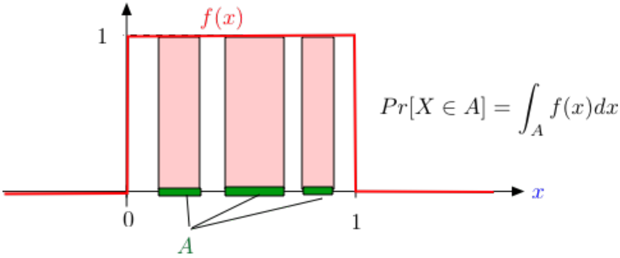
Then $Pr[X \in A]$ is the probability mass over A.

Uniformly at Random in $[0,1]$.



Think of $f(x)$ as describing how
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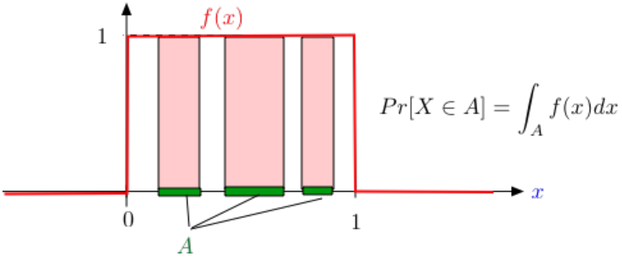
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
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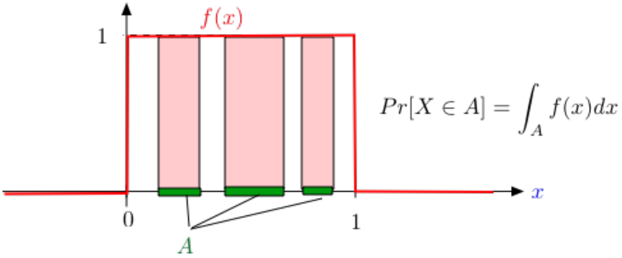
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.

Uniformly at Random in $[0, 1]$.



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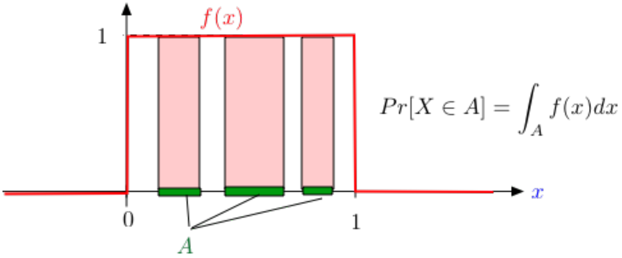


Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$: uniformly!

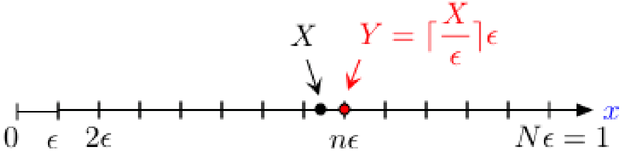
Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.
- We need $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

Uniformly at Random in $[0, 1]$.

Uniformly at Random in $[0, 1]$.



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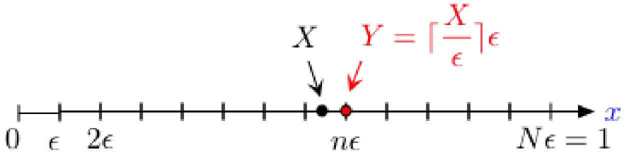
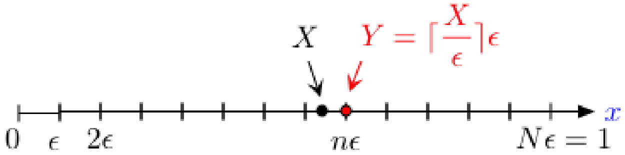


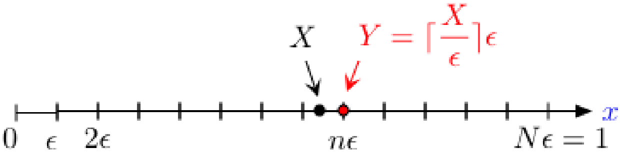
Discrete Approximation:

Uniformly at Random in $[0, 1]$.



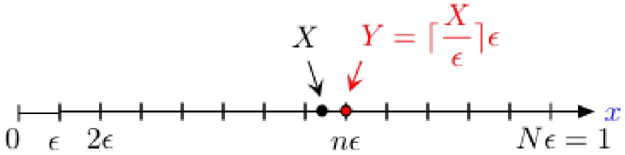
Discrete Approximation: Fix $N \gg 1$

Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

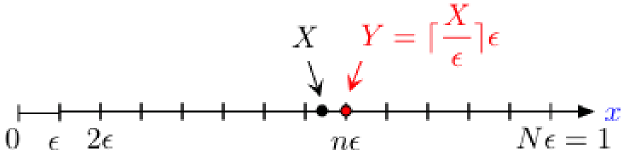
Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Uniformly at Random in $[0, 1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

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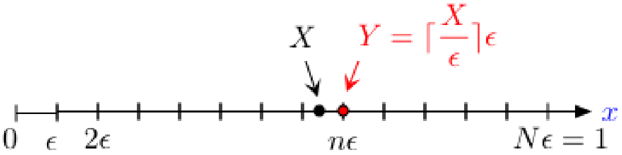
Then $|X - Y| \leq \varepsilon$

Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

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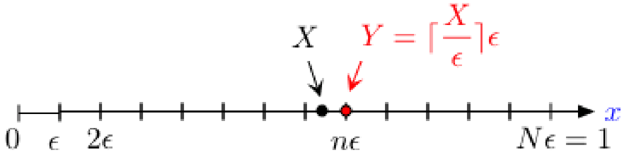
Then $|X - Y| \leq \varepsilon$ and Y is discrete:

Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

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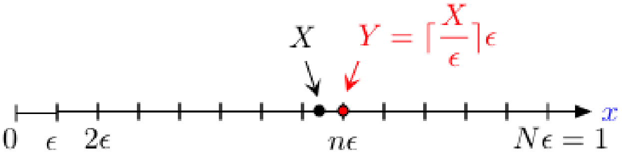

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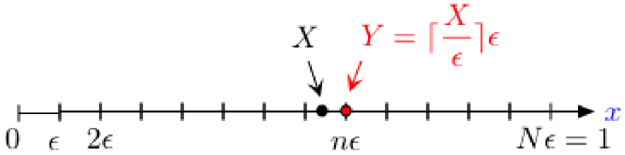

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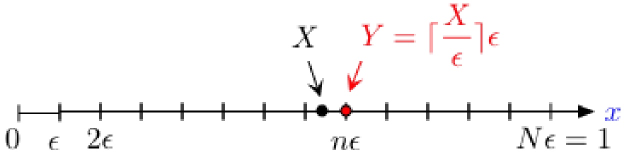

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Calculus view: $Pr[Y = n\varepsilon]$ is area of rectangle in Riemann sum.

Nonuniformly at Random in $[0, 1]$.

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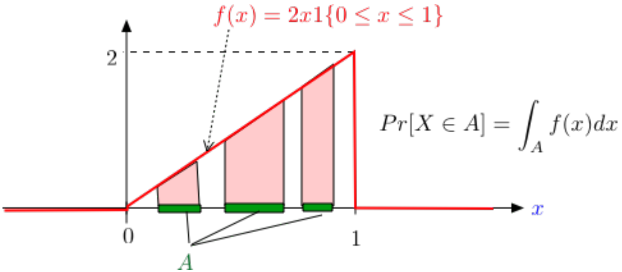

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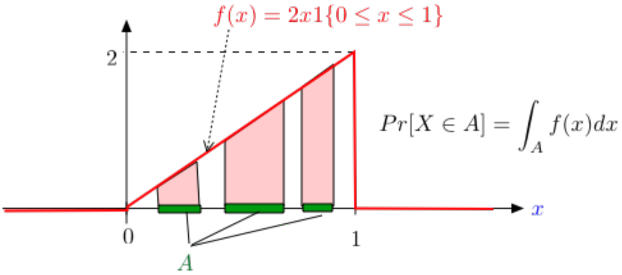

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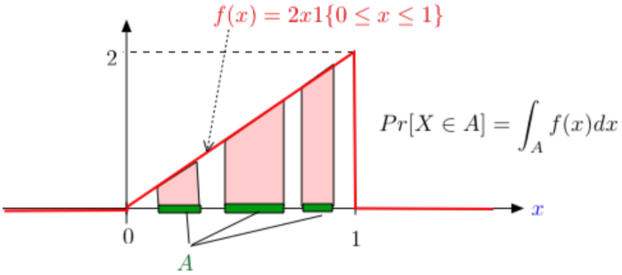

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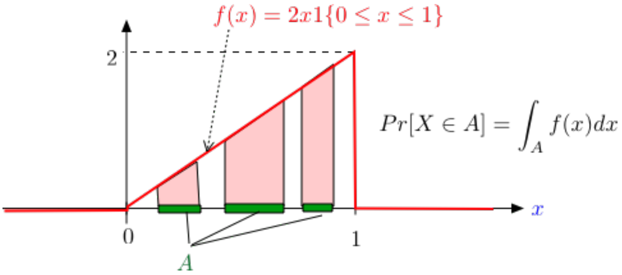

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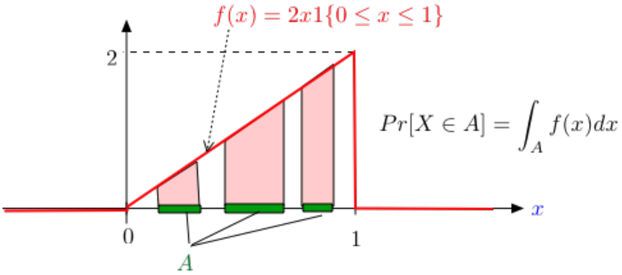

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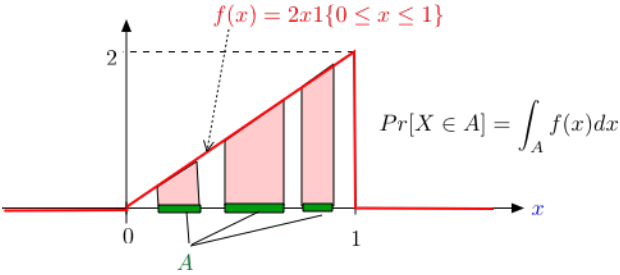

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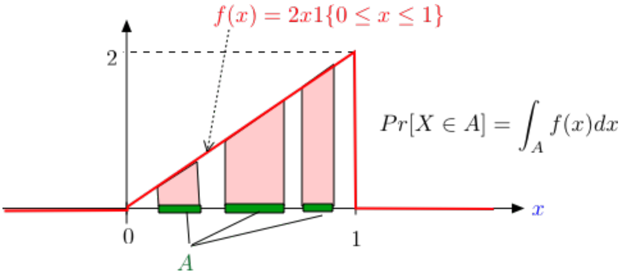

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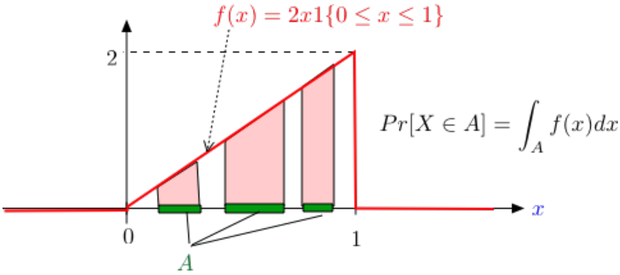

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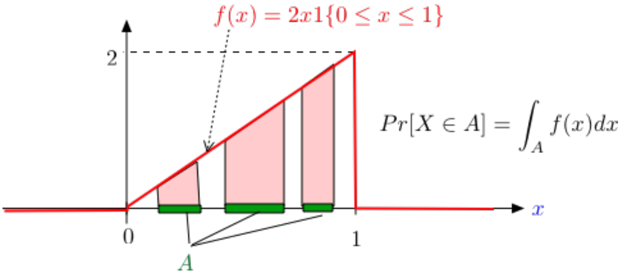

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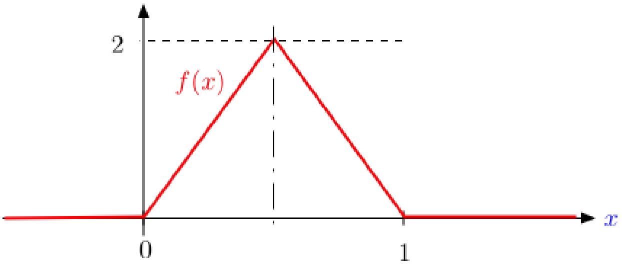

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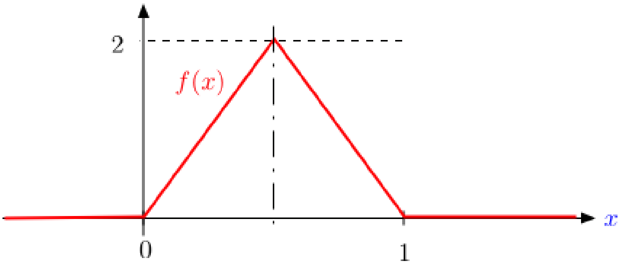

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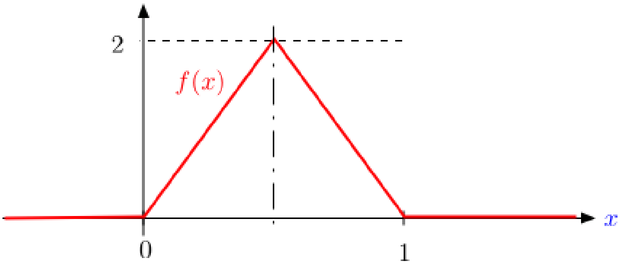

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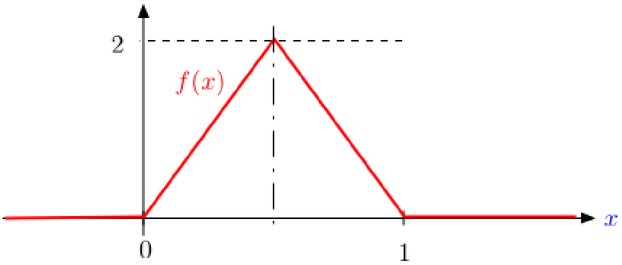

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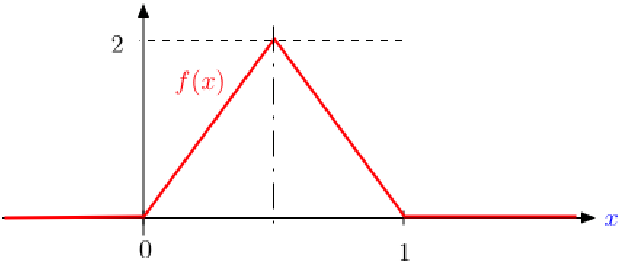

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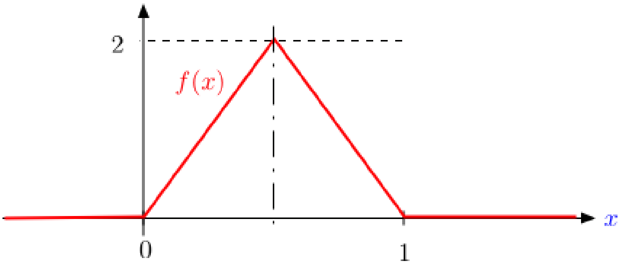

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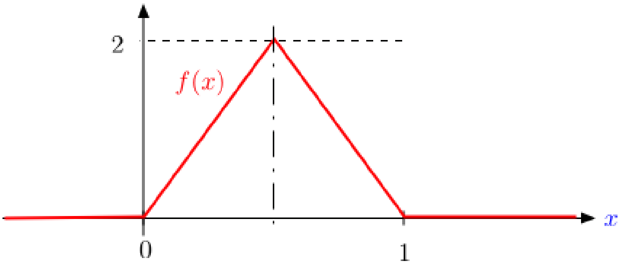

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For instance, $Pr[X \in [0, 1/3]] = \int_0^{1/3} 4xdx = 2\left[x^2\right]_0^{1/3} = \frac{2}{9}$.

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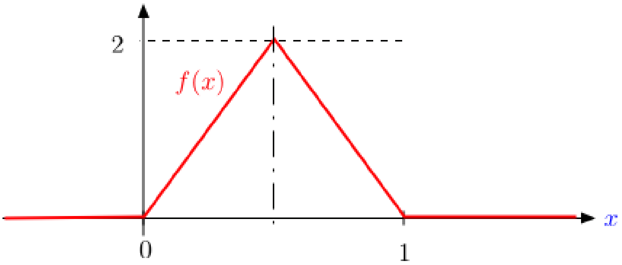

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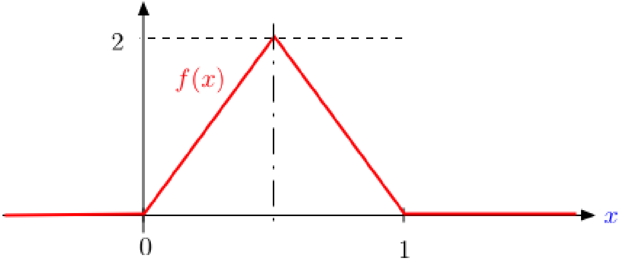

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Thus, $Pr[X \in [0, 1/3]] = Pr[X \in [2/3, 1]] = \frac{2}{9}$ and $Pr[X \in [1/3, 2/3]] =$

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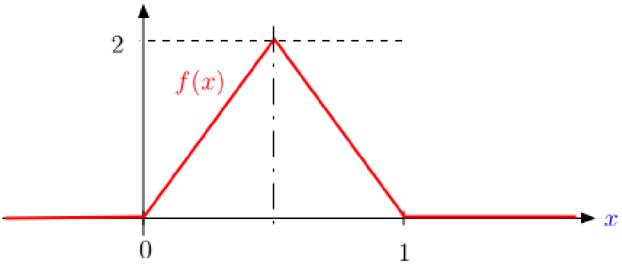

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Thus, $Pr[X \in [0, 1/3]] = Pr[X \in [2/3, 1]] = \frac{2}{9}$ and $Pr[X \in [1/3, 2/3]] = \frac{5}{9}$.

General Random Choice in $\Re$

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Let $F(x)$ be a nondecreasing function

General Random Choice in $\mathfrak{R}$

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$$
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$f(x)$ is the probability density function (pdf) of X.

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When F and f correspond RV X:

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$F(x)$ is cumulative distribution function (cdf) of X

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When F and f correspond RV X: $F_X(x)$ and $f_X(x)$.

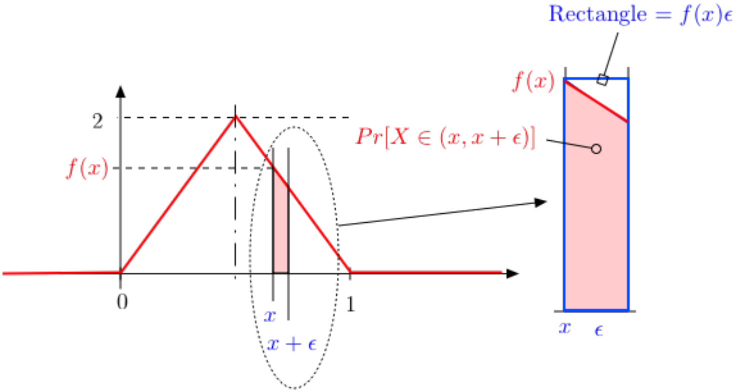
$Pr[X \in (x, x+\varepsilon)]$

$Pr[X \in (x, x+\varepsilon)]$

An illustration of $Pr[X \in (x, x+\varepsilon)] \approx f_X(x)\varepsilon$:

$Pr[X \in (x, x+\varepsilon)]$

An illustration of $Pr[X \in (x, x+\varepsilon)] \approx f_X(x)\varepsilon$:



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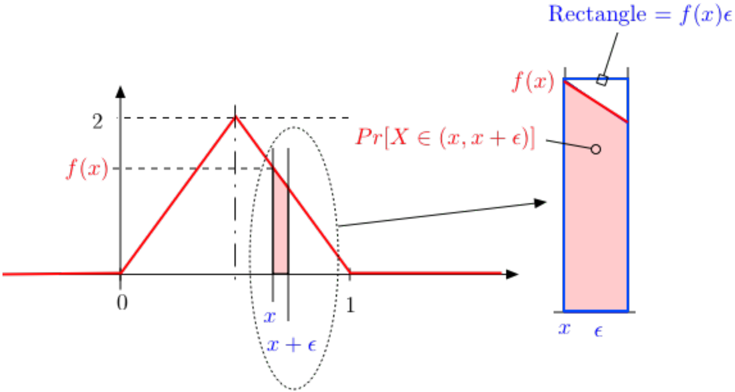

Thus, the pdf is the 'local probability by unit length.'

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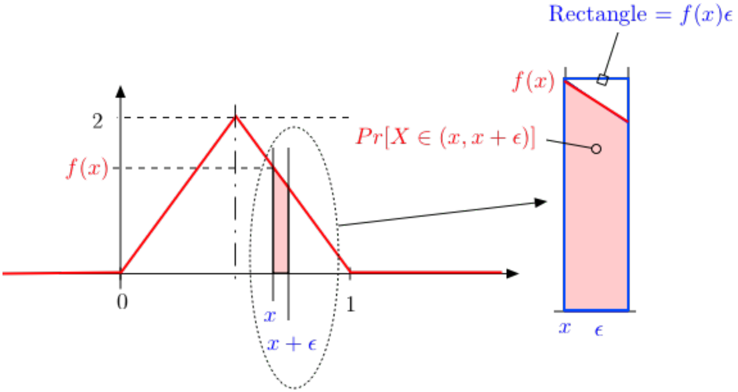

Thus, the pdf is the ‘local probability by unit length.’

It is the ‘probability density.’

Discrete Approximation

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Fix $\varepsilon \ll 1$

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Example: CDF, pre-poll

Example: hitting random location on gas tank.

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Example: hitting random location on gas tank.
Random location on circle.

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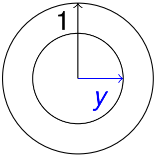

What is probability of being within y of the center, for non-negative $y \leq 1$?

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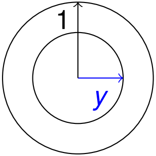

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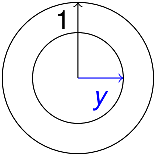

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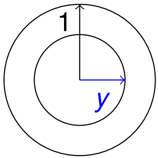

Random Variable: Y distance from center.

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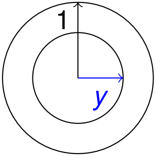

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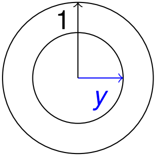

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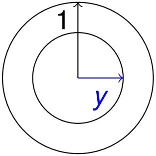

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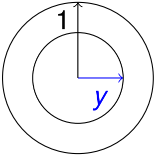

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Hence,

$$
F_Y(y) = Pr[Y \leq y] = \begin{cases} 0 & \text{for } y < 0 \\ y^2 & \text{for } 0 \leq y \leq 1 \\ 1 & \text{for } y > 1 \end{cases}
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Calculation of event with dartboard..

Probability between .5 and .6 of center?

Calculation of event with dartboard..

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$$Pr[0.5 < Y \leq 0.6] \quad = \quad Pr[Y \leq 0.6] - Pr[Y \leq 0.5]$$

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PDF.

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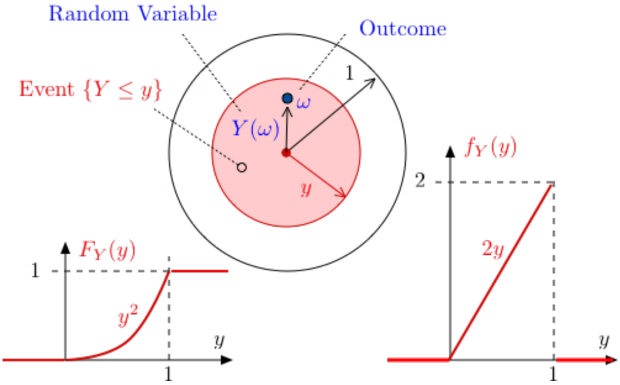
The cumulative distribution function (cdf) and probability distribution function (pdf) give full information.

Use whichever is convenient.

Target

Target



Random Variable
Outcome
Event $\{Y \leq y\}$
1
ω
$Y(\omega)$
y
$f_Y(y)$
2
$2y$
y
1
$F_Y(y)$
1
y^2
y
1

$U[a, b]$

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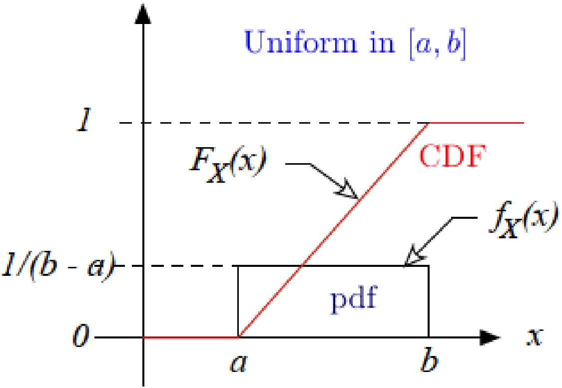

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(A) $X \sim G(p)$
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(C) $Pr[Y>i/n]=(1-\lambda/n)^i$.
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$Pr[Y > y]$ is defined as "Survival function."

$Expo(\lambda)$

"Limit of geometric."

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From last slide: $S(t) = Pr[X > t] = e^{-\lambda t}$ for $t > 0$.
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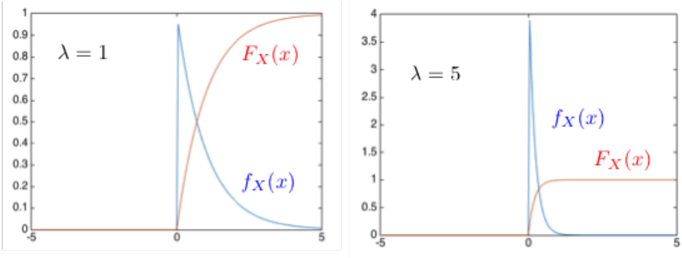

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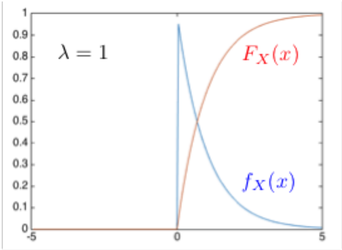

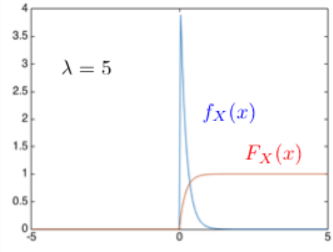

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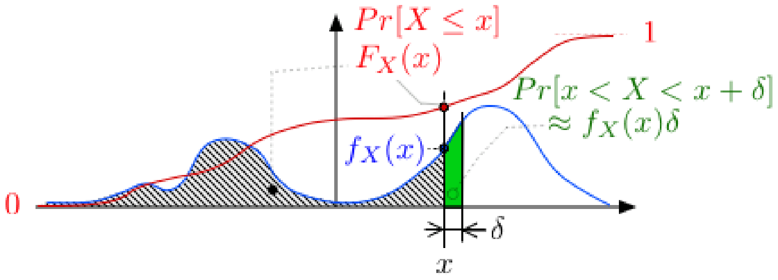
A Picture



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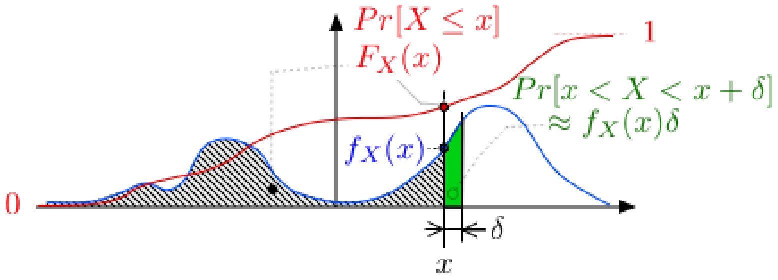

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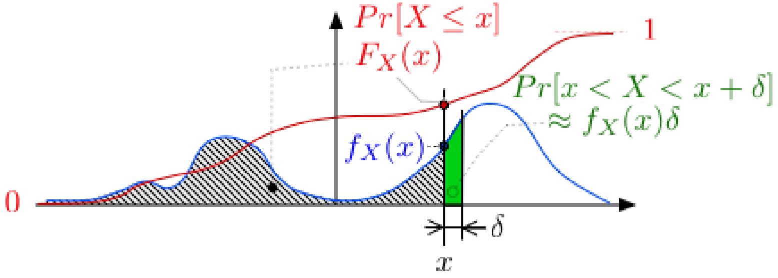

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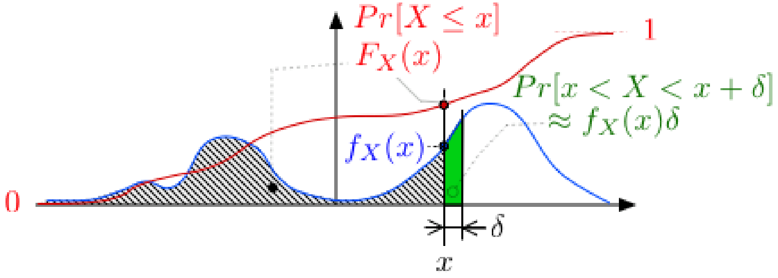

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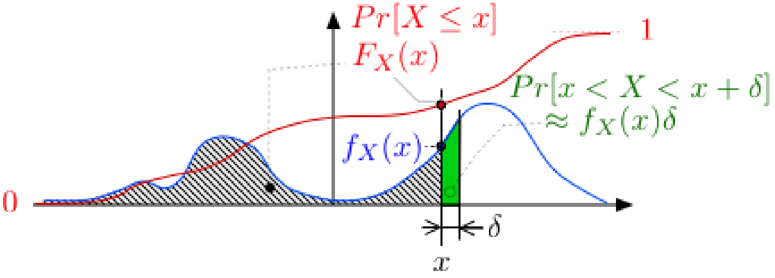

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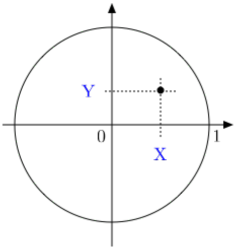

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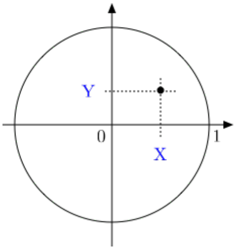

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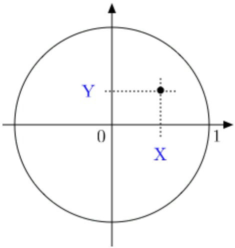

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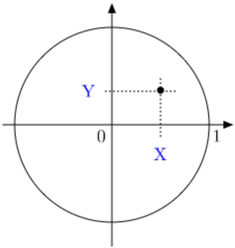

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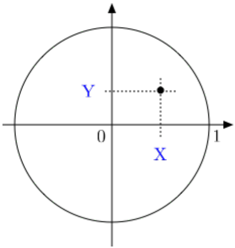

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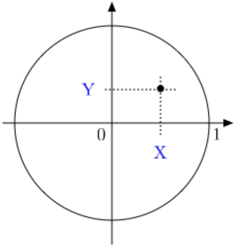

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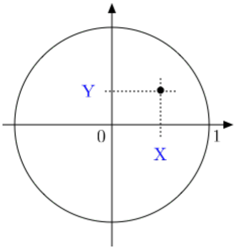

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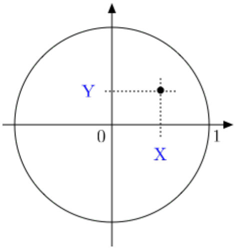

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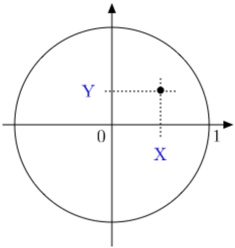

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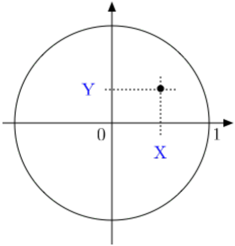

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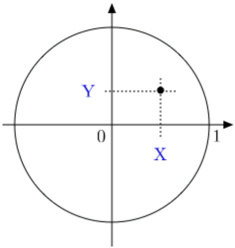

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Thus, $f_{X,Y}(x,y) = f_X(x)f_Y(y)$.

$\square$

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

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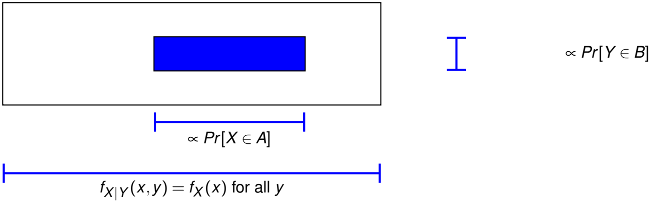

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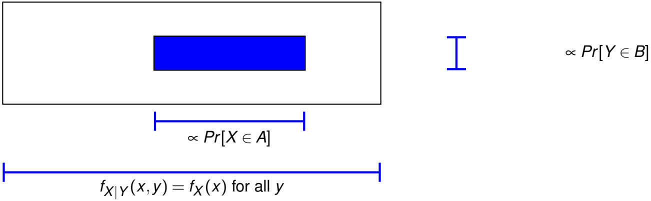

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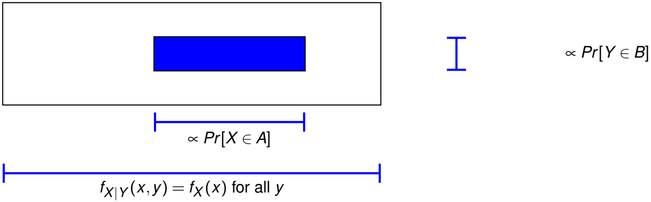

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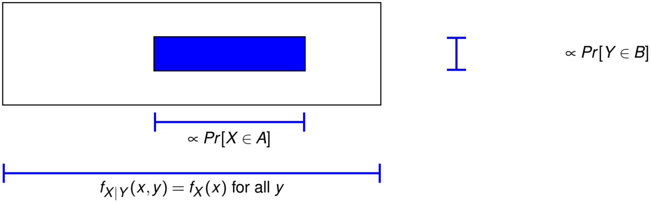

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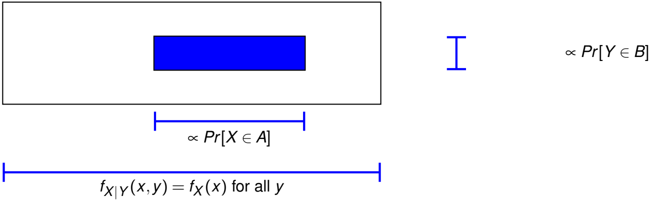

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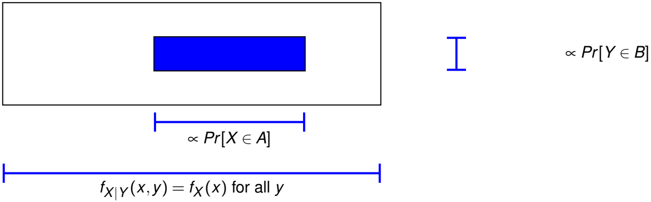

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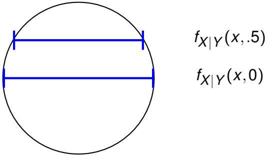

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