Stars and Bars Poll

Mark whats correct.

(A) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(B) ways to split k dollars among n: $\binom{k+n-1}{n-1}$

(C) ways to split 5 dollars among 3: $\binom{7}{5}$

(D) ways to split 5 dollars among 3: $\binom{5+3-1}{3-1}$

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All correct.

Poll

Mark whats corect.

Poll

Mark whats corect.

(A) $|10 \text{ digit numbers}| = 10^{10}$
(B) $|k \text{ coin tosses}| = 2^k$
(C) $|10 \text{ digit numbers}| = 9 * 10^9$
(D) $|n \text{ digit base } m \text{ numbers}| = m^n$
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(A) or (C)? (D) or (E)? (B) are correct.

Refresh: Counting.

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Add number of each

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Disjoint – so add!

CS70: On to probability.

$\boxed{\text{Modeling Uncertainty: Probability Space}}$

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Modeling Uncertainty: Probability Space

1. Key Points
2. Random Experiments
3. Probability Space

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  - Models knowledge about uncertainty
  - Optimizes use of knowledge to make decisions

The Magic of Probability

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Uncertainty:

The Magic of Probability

Uncertainty: vague,

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Precise,

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Your cost: focused attention and practice on examples and problems.

Random Experiment: Flip one Fair Coin

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What do we mean by the likelihood of tails is 50%?

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Flip a fair coin: model

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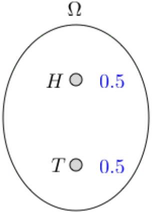
Physical Experiment

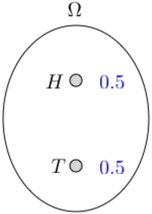


Probability Model

Random Experiment: Flip one Fair Coin

Flip a fair coin: model

Physical Experiment



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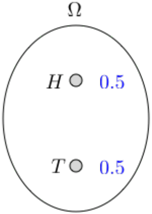

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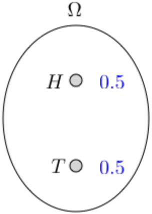

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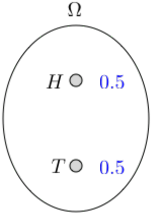

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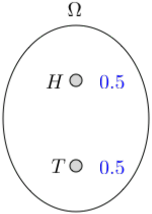

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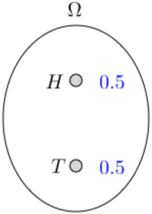

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- Tautology? No: Statistical regularity!

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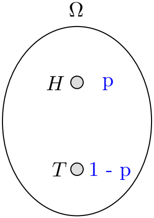

Probability Model

Flip Two Fair Coins

Flip Two Fair Coins

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Flip Two Fair Coins

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Flips two coins glued together side by side:

Flip Glued Coins

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Glued coins


50%


50%

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Glued coins

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- Note: Coins are glued so that they show different faces.

Flip two Attached Coins

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Flips two coins attached by a spring:

Flip two Attached Coins

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- Note: Coins are attached so that they tend to show the same face, unless the spring twists enough.

Flipping Two Coins

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Here is a way to summarize the four random experiments:

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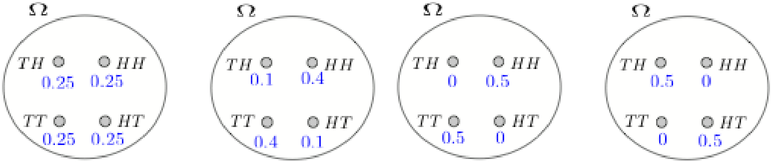

[1] [2] [3] [4]

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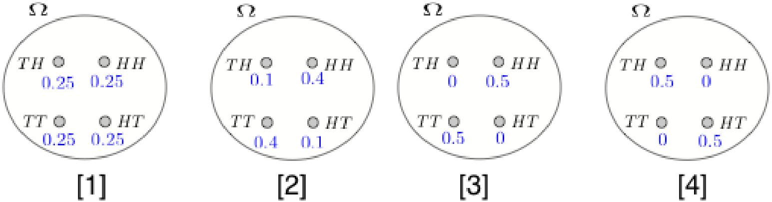

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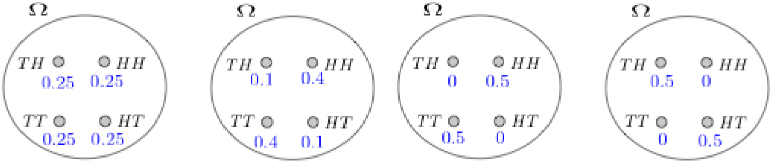

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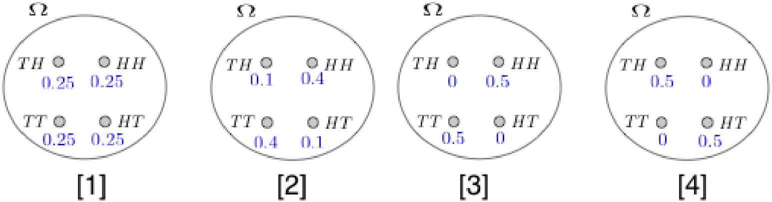

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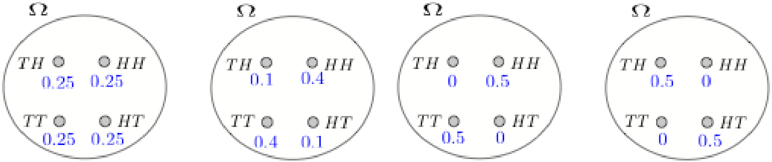

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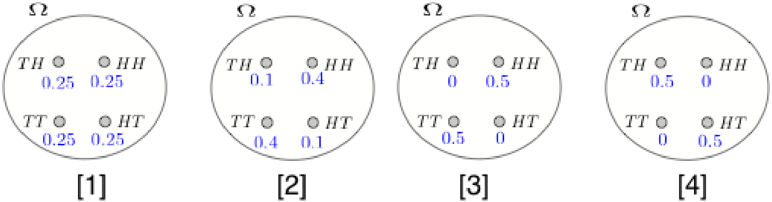

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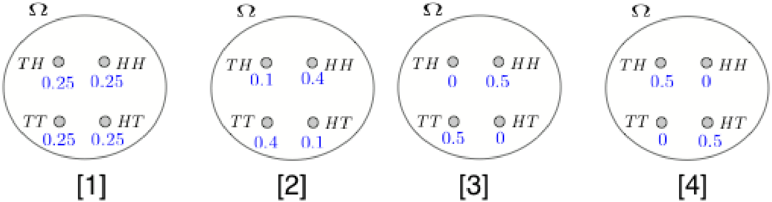

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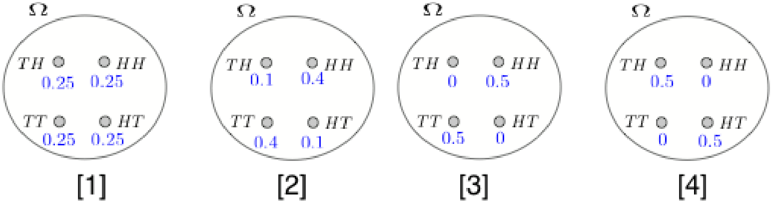

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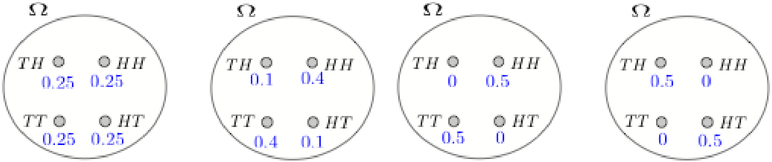

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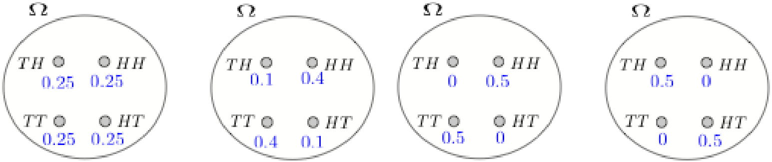

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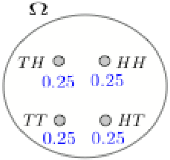

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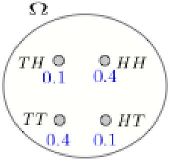

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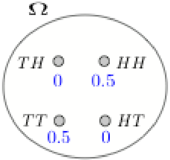

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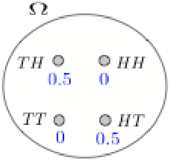
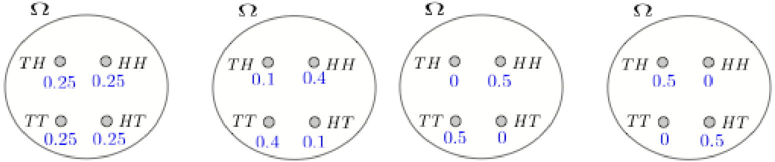


[4]

Flipping Two Coins



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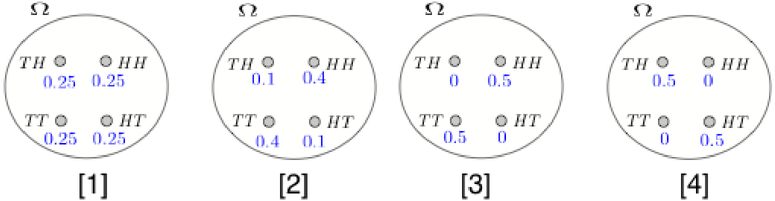
Flipping Two Coins



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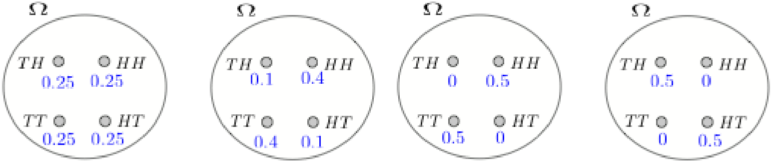

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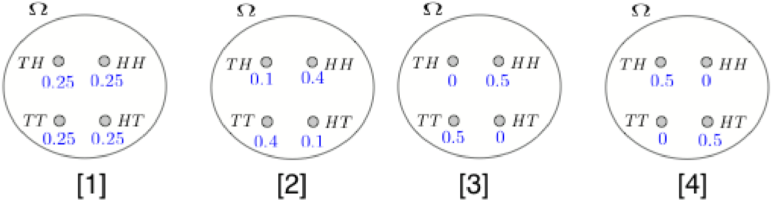

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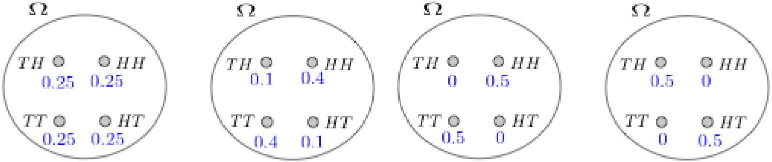

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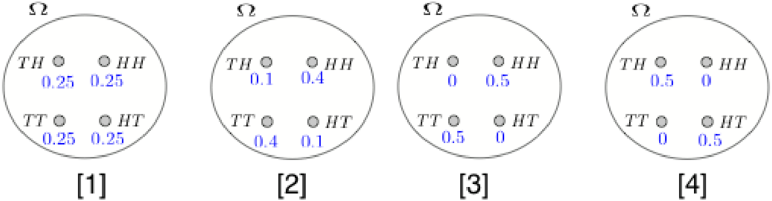

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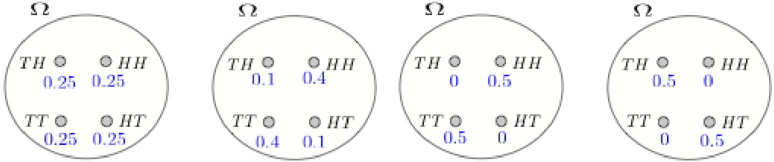

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Flipping n times

Flip a fair coin n times (some $n \geq 1$):

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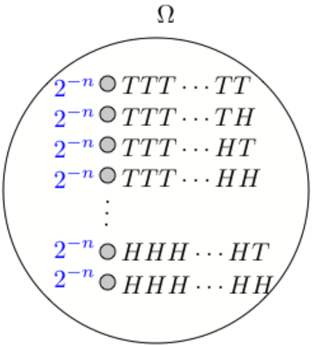

Roll two Dice

Roll a balanced 6-sided die twice:

Roll two Dice

Roll a balanced 6-sided die twice:

- Possible outcomes:

Roll two Dice

Roll a balanced 6-sided die twice:

- Possible outcomes: $\{1,2,3,4,5,6\}^2 = \{(a,b) \mid 1 \leq a,b \leq 6\}$.

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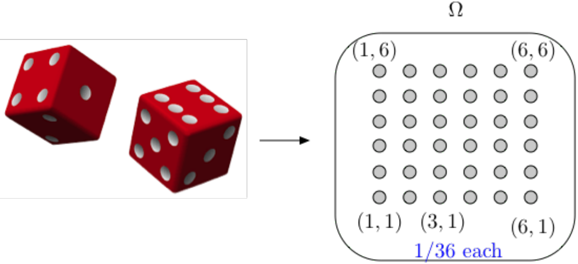

Physical Experiment   Probability Model

Probability Space.

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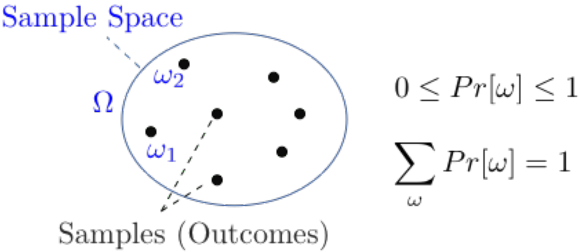

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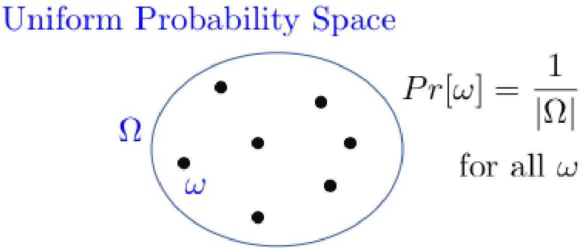

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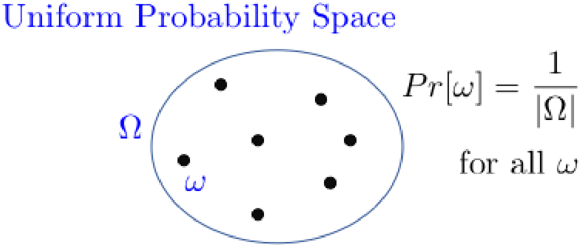

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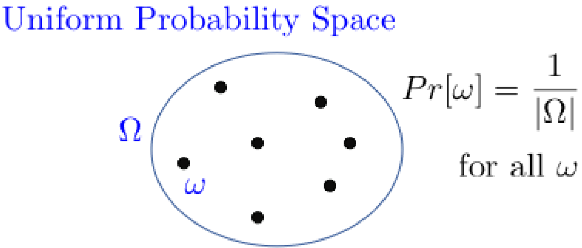

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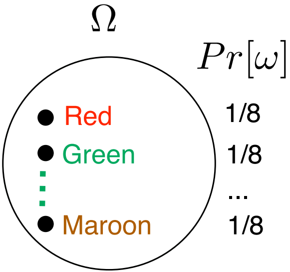
Simplest physical model of a uniform probability space:

Probability Space: Formalism

Simplest physical model of a uniform probability space:

Physical experiment



Probability model

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Simplest physical model of a uniform probability space:

Physical experiment

Ω $\quad Pr[\omega]$

- Red 1/8
- Green 1/8
- ⋮ ...
- Maroon 1/8

Probability model

A bag of identical balls, except for their color (or a label).

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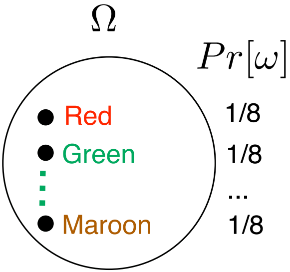

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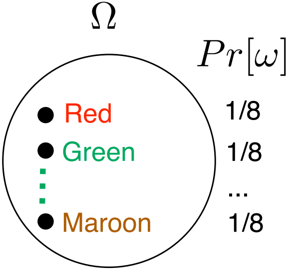

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A bag of identical balls, except for their color (or a label). If the bag is well shaken, every ball is equally likely to be picked.

$$\Omega = \{\text{white, red, yellow, grey, purple, blue, maroon, green}\}$$

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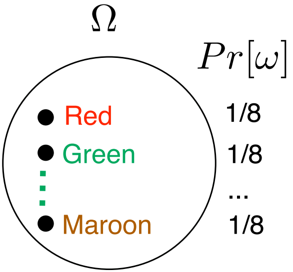

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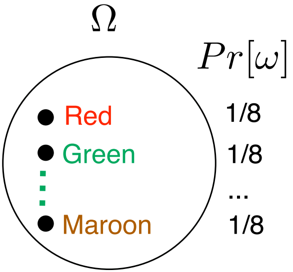

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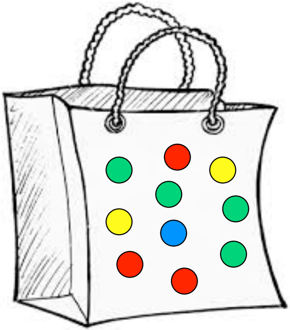

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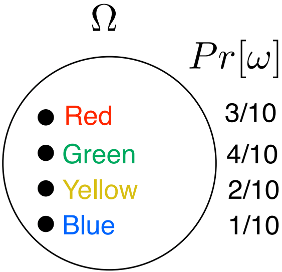

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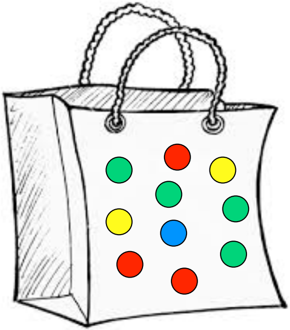

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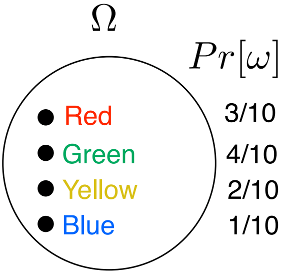

Probability model

$\Omega = \{$Red, Green, Yellow, Blue$\}$

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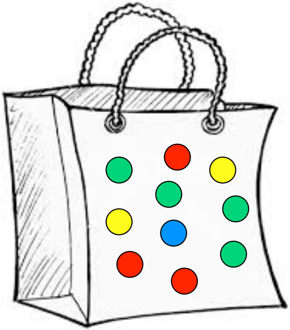

Physical experiment

Ω	$Pr[\omega]$
Red	3/10
Green	4/10
Yellow	2/10
Blue	1/10

Probability model

$$\Omega = \{\text{Red, Green, Yellow, Blue}\}$$

$$Pr[\text{Red}] =$$

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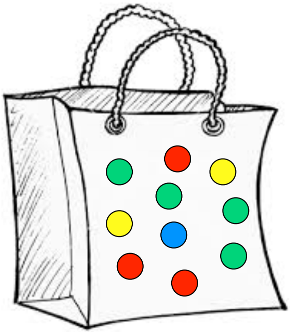

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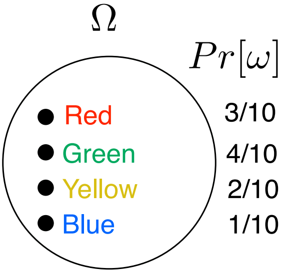

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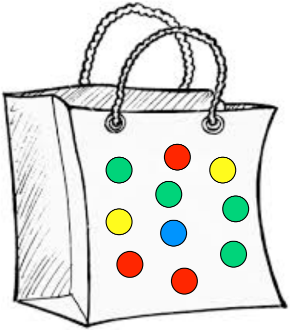

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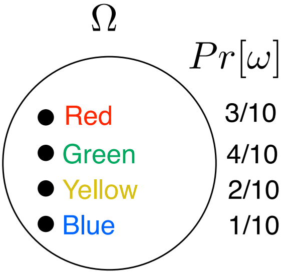

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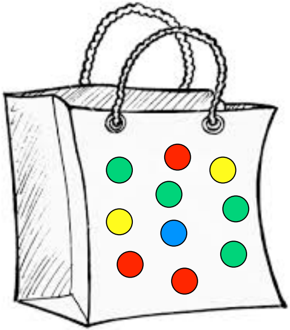

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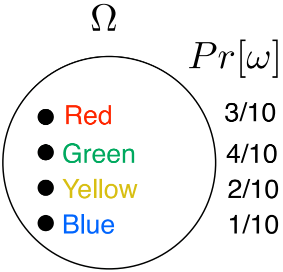

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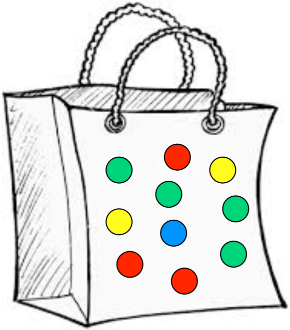

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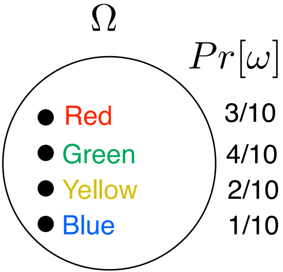

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Note: Probabilities are restricted to rational numbers: $\frac{N_k}{N}$.

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Physical model of a general non-uniform probability space:

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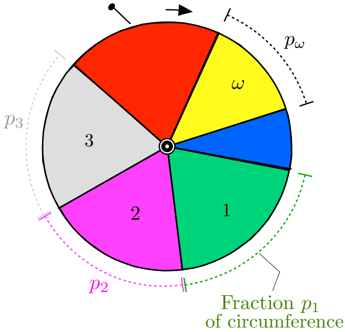

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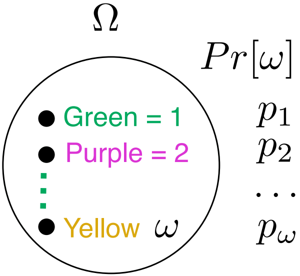

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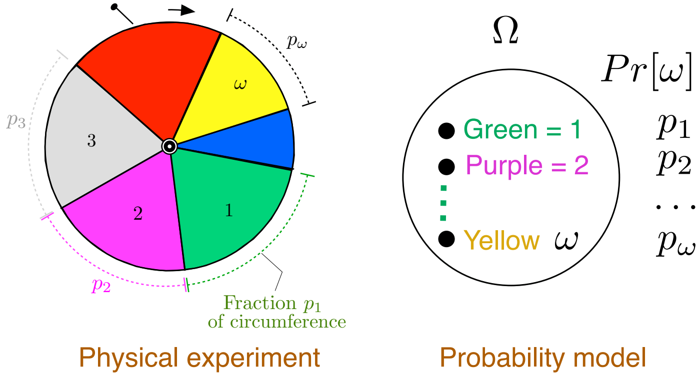

The roulette wheel stops in sector ω with probability p_ω.

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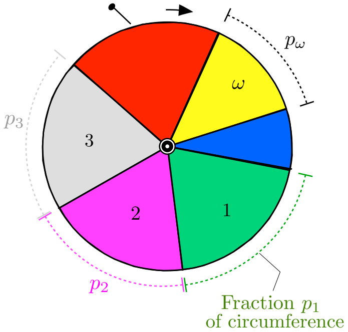

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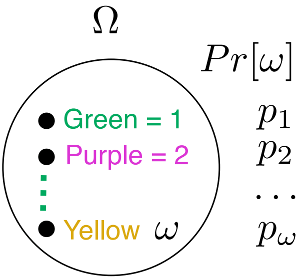

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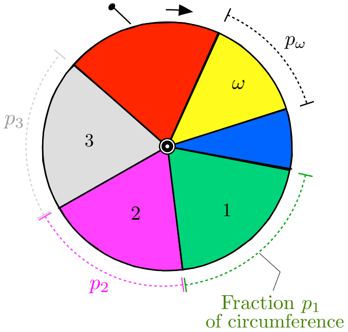

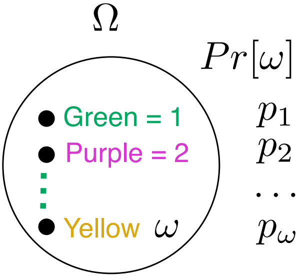

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  - The experiment selects *one* of the elements of Ω.
- In this case, its wrong to think that $\Omega = \{H, T\}$ and that the experiment selects two outcomes.
- Why? Because this would not describe how the two coin flips are related to each other.
- For instance, say we glue the coins side-by-side so that they face up the same way. Then one gets *HH* or *TT* with probability 50% each. This is not captured by 'picking two outcomes.'

Summary of Probability Basics

Modeling Uncertainty: Probability Space

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1. Random Experiment

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$\boxed{\text{Events, Conditional Probability, Independence, Bayes' Rule}}$

CS70: On to Events.

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Today: Events.

Probability Basics Review

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Set notation review

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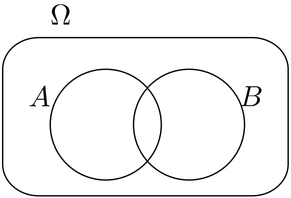
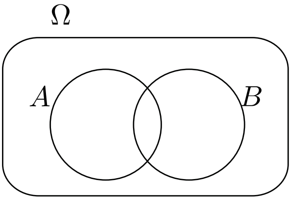


Figure: Two events

Set notation review



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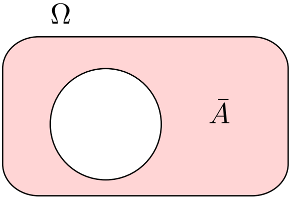
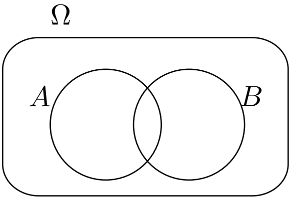


Figure: Complement (not)

Set notation review



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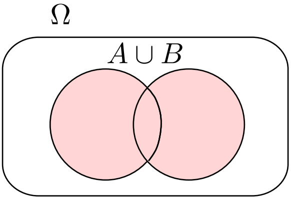

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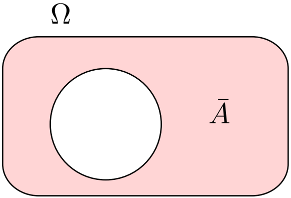

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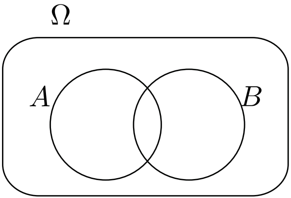

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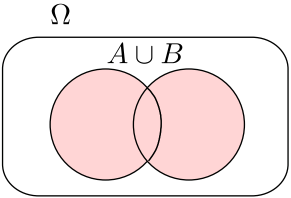

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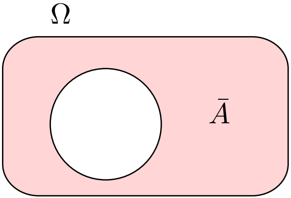

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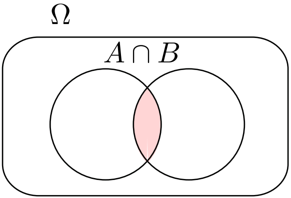

Figure: Intersection (and)

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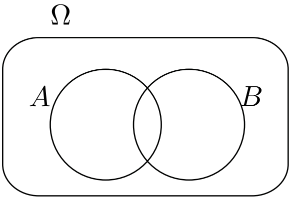

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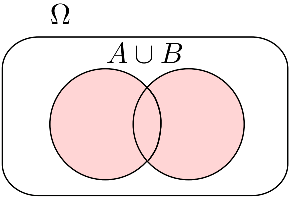

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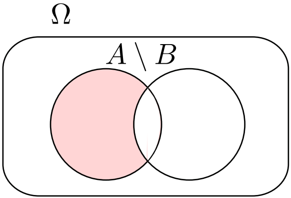

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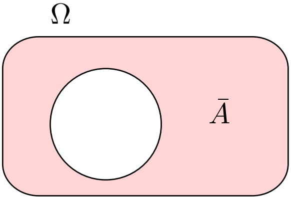

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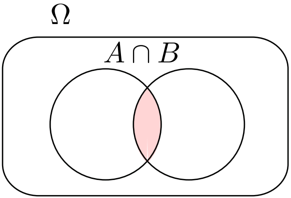

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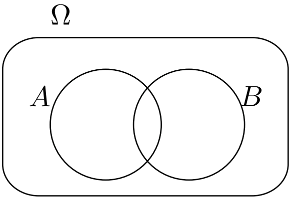

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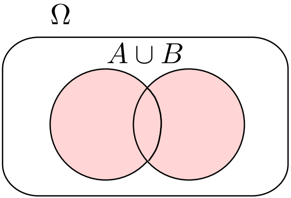

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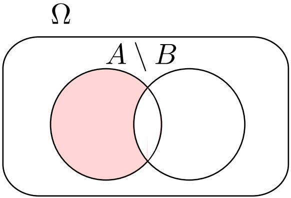

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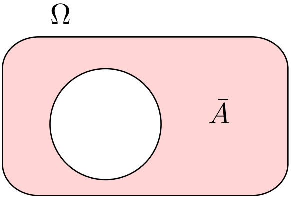

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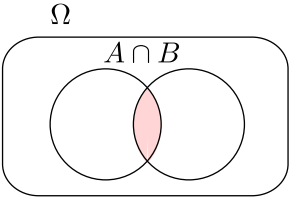

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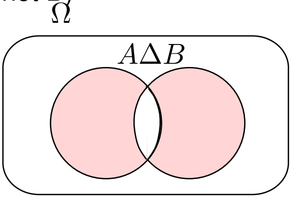

Figure: Symmetric difference (only one)

Probability of exactly one ‘heads’ in two coin flips?

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Idea: Sum the probabilities of all the different outcomes that have exactly one ‘heads’: HT, TH.

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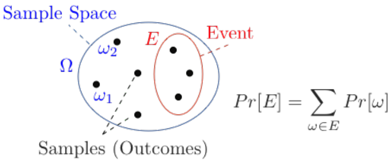

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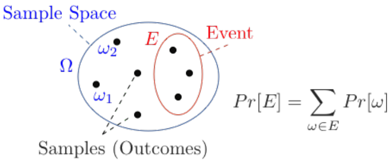

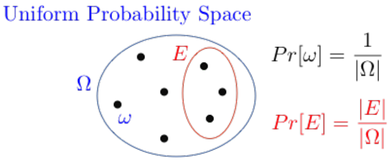

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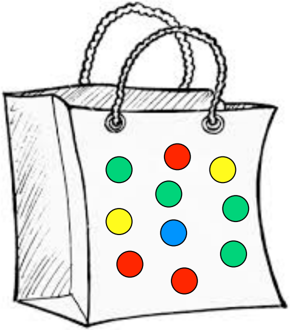

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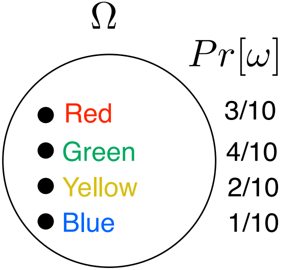
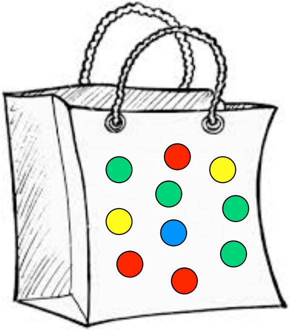


Probability model

Event: Example



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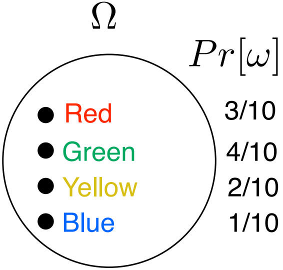

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$\Omega =$ {Red, Green, Yellow, Blue}

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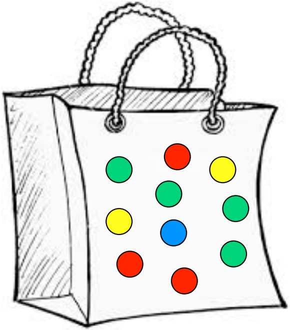

Physical experiment

Ω

	$Pr[\omega]$
Red	3/10
Green	4/10
Yellow	2/10
Blue	1/10

Probability model

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$Pr[\text{Red}] =$

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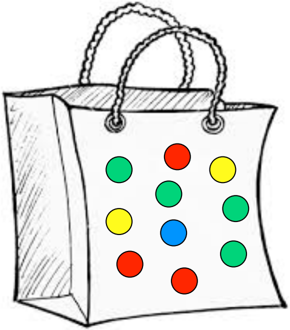

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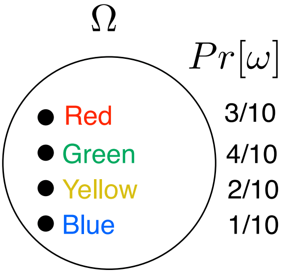

Probability model

$$\Omega = \{\text{Red, Green, Yellow, Blue}\}$$
$$Pr[\text{Red}] = \frac{3}{10},$$

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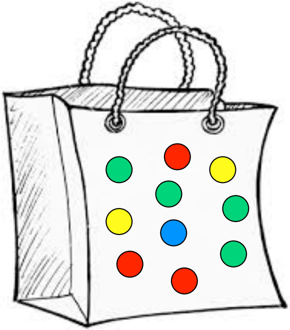

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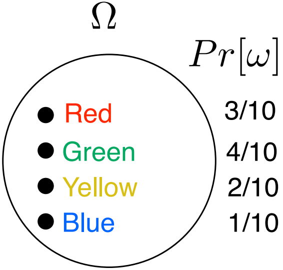

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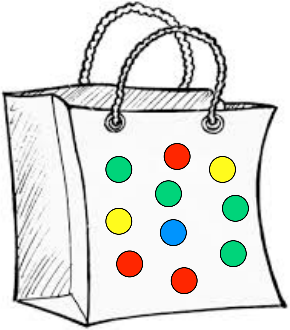

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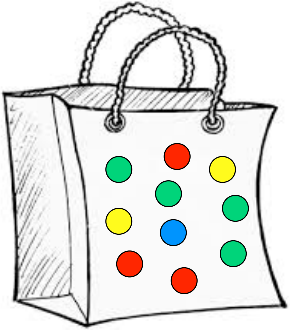

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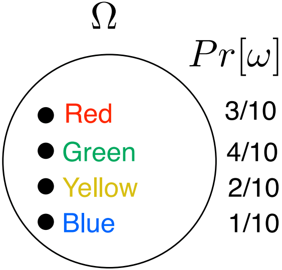

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$E = \{Red, Green\}$

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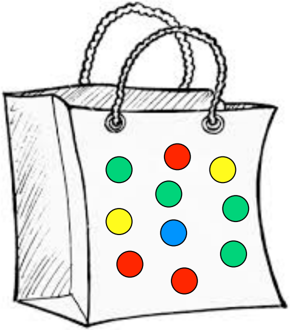

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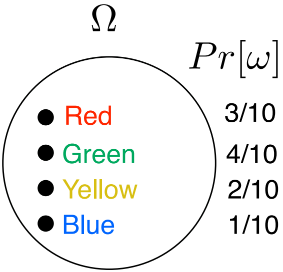

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$E = \{Red, Green\} \Rightarrow Pr[E] =$

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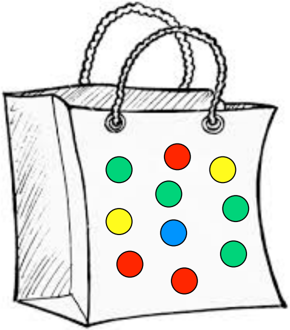

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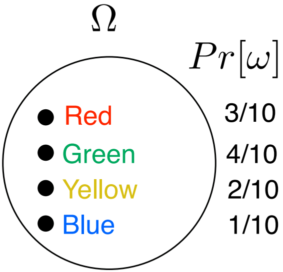

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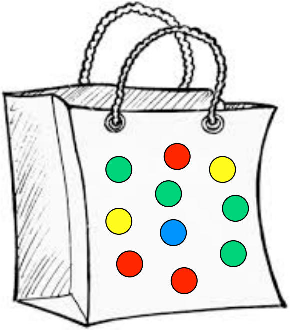

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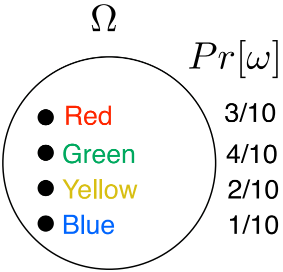

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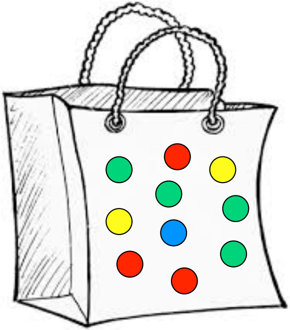

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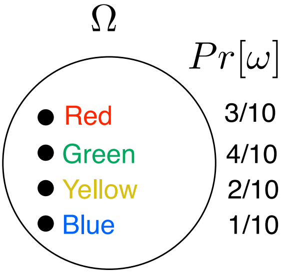

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Uniform probability space: $Pr[HH] = Pr[HT] = Pr[TH] = Pr[TT] = \frac{1}{4}$.

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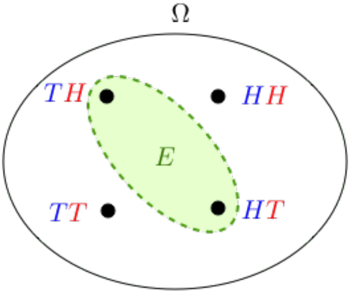

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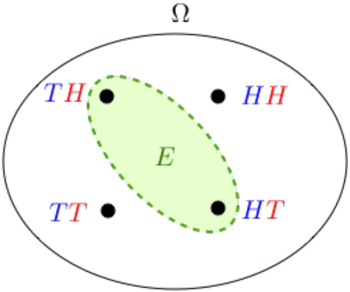


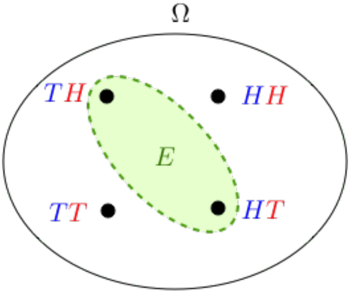
$$Pr[E] = \sum_{\omega \in E} Pr[\omega]$$

Probability of exactly one heads in two coin flips?

Sample Space, $\Omega = \{HH, HT, TH, TT\}$.

Uniform probability space: $Pr[HH] = Pr[HT] = Pr[TH] = Pr[TT] = \frac{1}{4}$.

Event, E, "exactly one heads": $\{TH, HT\}$.



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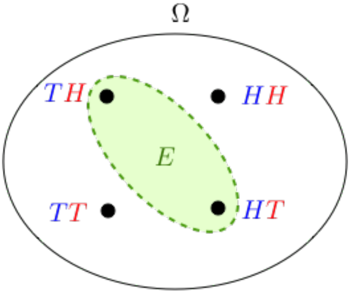

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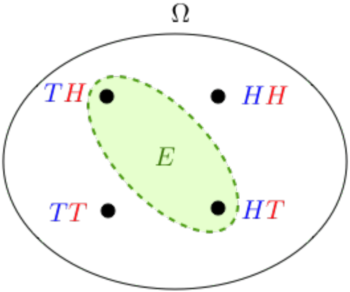

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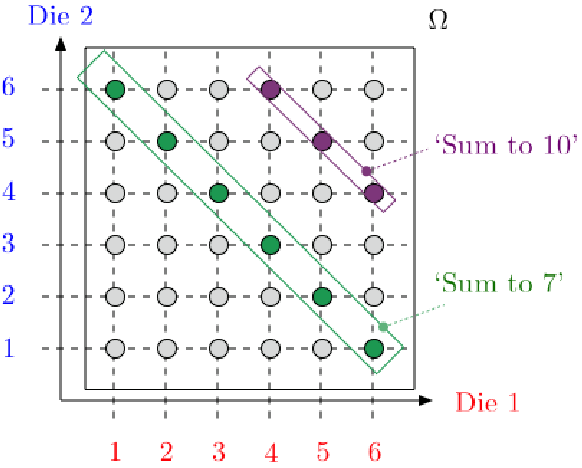

$$Pr[\text{Sum to 7}] = \frac{6}{36} \qquad Pr[\text{Sum to 10}] = \frac{3}{36}$$

Example and Polls: 20 coin tosses.

20 coin tosses

Sample space: $\Omega =$ set of 20 fair coin tosses.

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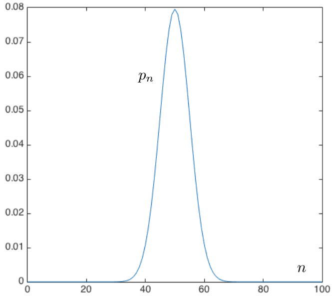

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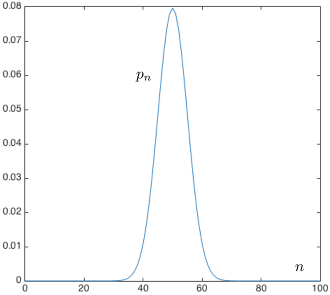

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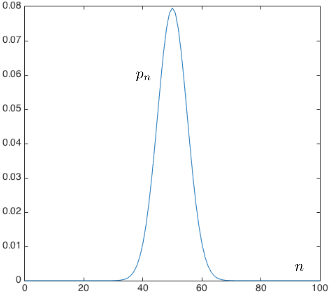

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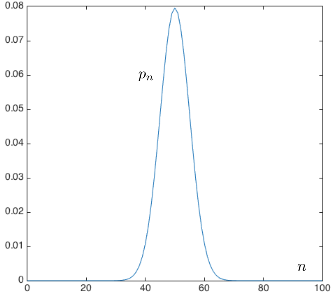

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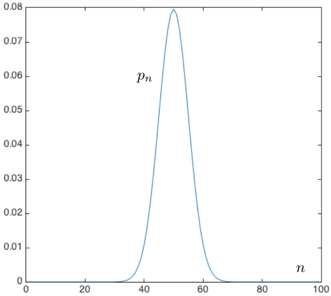

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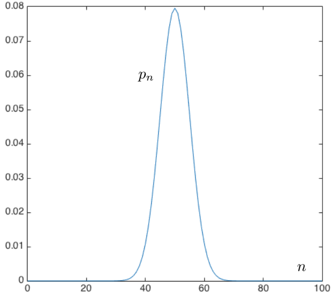

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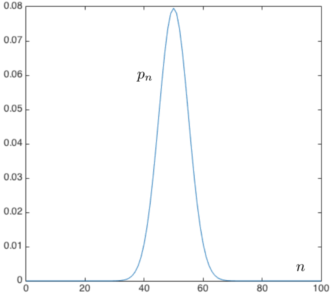

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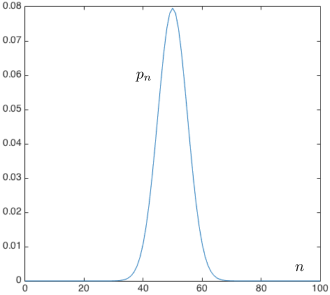

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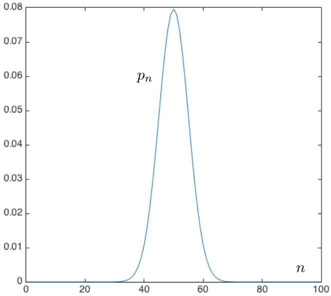

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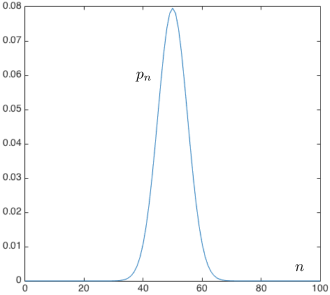

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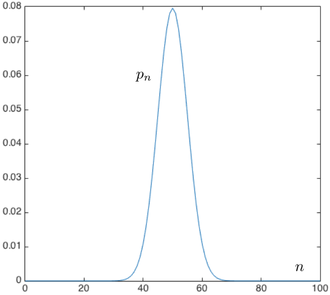

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$p_n := Pr[E_n] = \frac{|E_n|}{|\Omega|} = \frac{\binom{100}{n}}{2^{100}}$

Observe:

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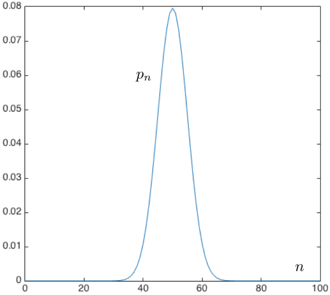
$\Omega = \{H, T\}^{100}$; $|\Omega| = 2^{100}$.



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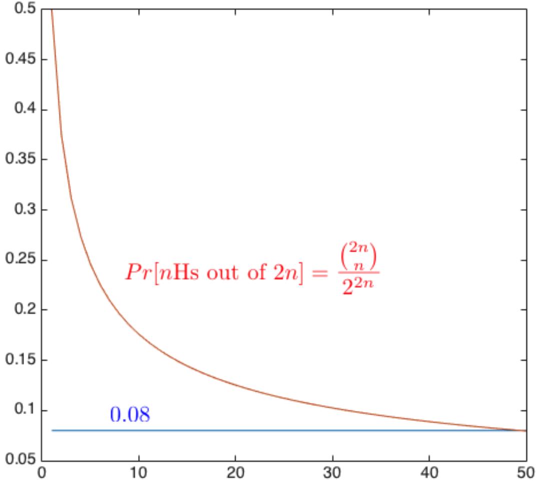

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4. Event: "subset of outcomes." $A \subseteq \Omega$. $Pr[A] = \sum_{w \in A} Pr[\omega]$
5. Some calculations.