CS70: Lecture 9. Outline.

1. Public Key Cryptography
2. RSA system
   2.1 Efficiency: Repeated Squaring.
   2.2 Correctness: Fermat's Theorem.
   2.3 Construction.
3. Warnings.

Isomorphisms.

Bijection:

Isomorphisms.

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$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Isomorphisms.

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Simplified Chinese Remainder Theorem:

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If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

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Now consider: $(a,b) + (a',b') = (0,2)$.

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65$

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

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Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$

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Try $43 + 22 = 65 = 20 \pmod{45}$.

Is it $0 \pmod{5}$? Yes! Is it $2 \pmod{9}$? Yes!

Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$
correspond to actions in $\pmod{45}$!

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

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What's true?

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

(A) $x + y = 2 \mod 7$
(B) $x + y = 2 \mod 6$
(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$
(F) $y = 39 \mod 42$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

What's true?

(A) $x + y = 2 \mod 7$
(B) $x + y = 2 \mod 6$
(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$
(F) $y = 39 \mod 42$

All true.

Xor

Computer Science:

Xor

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1 - True

0 - False

Xor

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$1 \vee 1 = 1$

Xor

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$A \oplus B$ - Exclusive or.

Xor

Computer Science:
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Note: Also modular addition modulo 2!

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Note: Also modular addition modulo 2!
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Property: $A \oplus B \oplus B = A$.

Xor

Computer Science:
1 - True
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Property: $A \oplus B \oplus B = A$.
By cases: $1 \oplus 1 \oplus 1 = 1$.

Xor

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Cryptography ...

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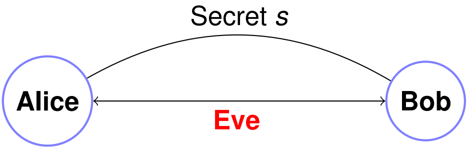

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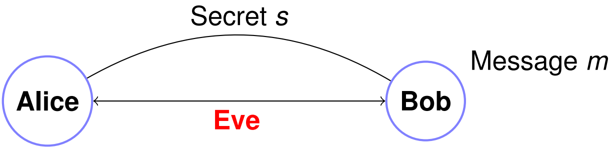

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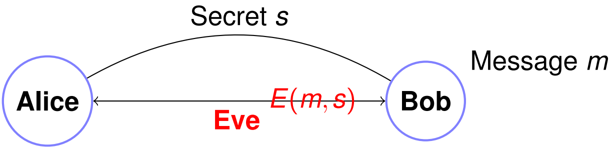

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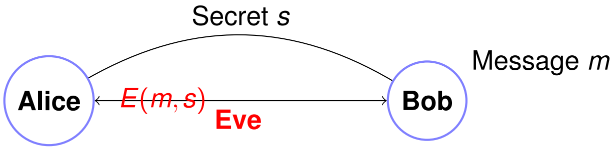

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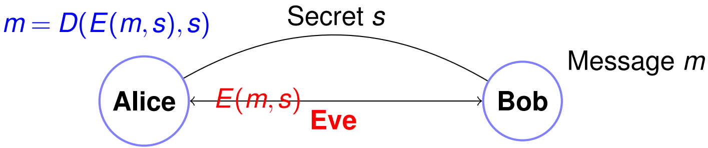

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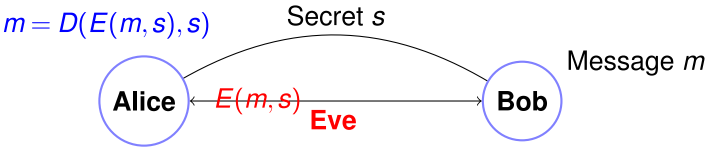
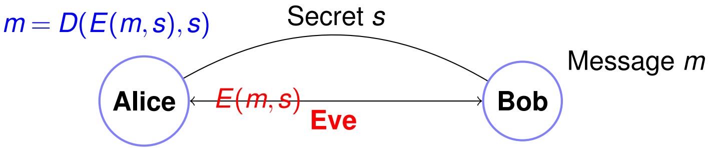


Example:

Cryptography ...



Example:
One-time Pad: secret s is string of length $|m|$.

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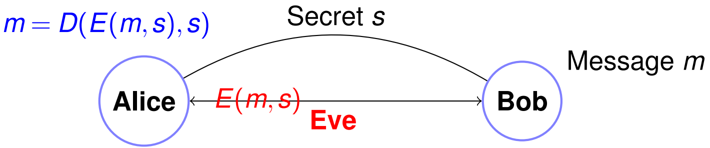
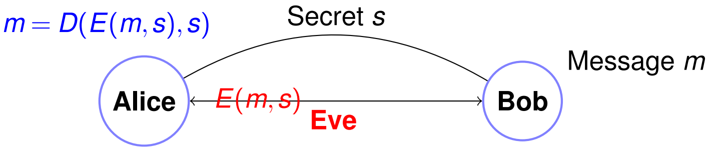


Example:

One-time Pad: secret s is string of length $|m|$.

$m = 1010101110101101$

Cryptography ...



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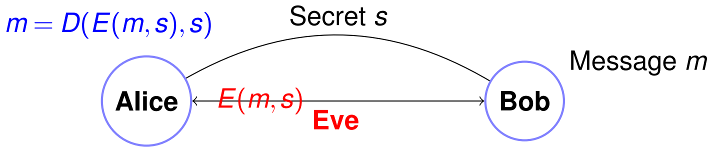


Example:

One-time Pad: secret s is string of length $|m|$.

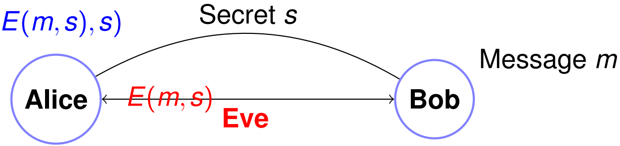
$m = 1010101111010110 1$

$s\ = ...$

$E(m,s)$ – bitwise $m \oplus s$.

Cryptography ...

$m = D(E(m,s),s)$



Example:

One-time Pad: secret s is string of length $|m|$.

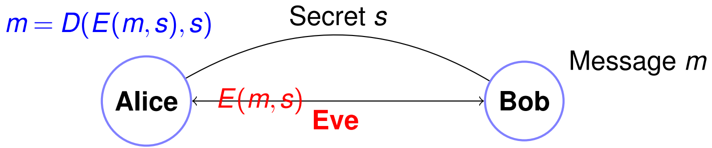
$m = 1010101110101101$

$s\ = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$E(m,s)$ – bitwise $m \oplus s$.

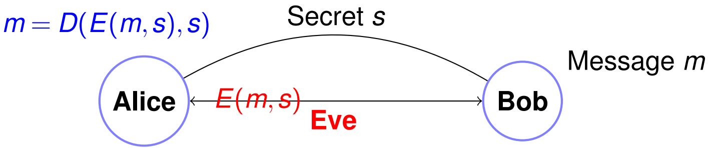
$D(x,s)$ – bitwise $x \oplus s$.

Cryptography ...



Example:
One-time Pad: secret s is string of length $|m|$.
$m = 1010101110101101$
$s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
$E(m,s)$ – bitwise $m \oplus s$.
$D(x,s)$ – bitwise $x \oplus s$.
Works because $m \oplus s \oplus s = m$!

Cryptography ...



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One-time Pad: secret s is string of length $|m|$.
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...and totally secure!

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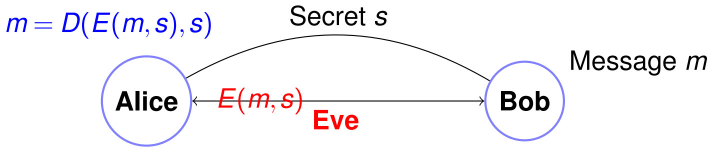

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...given $E(m,s)$ any message m is equally likely.

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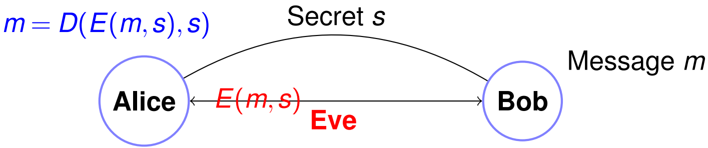

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Disadvantages:

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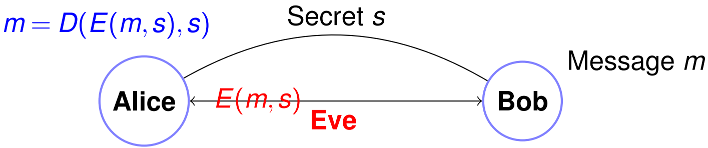

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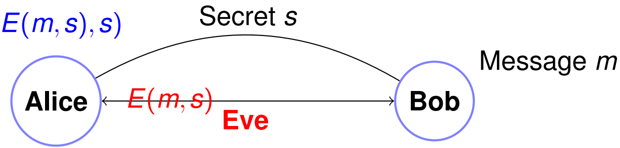
Disadvantages:

Shared secret!

Cryptography ...

$m = D(E(m,s),s)$



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$D(x,s)$ – bitwise $x \oplus s$.

Works because $m \oplus s \oplus s = m$!

...and totally secure!

...given $E(m,s)$ any message m is equally likely.

Disadvantages:

Shared secret!

Uses up one time pad..

Cryptography ...

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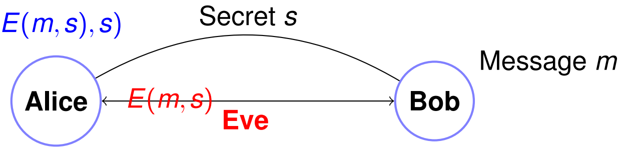

Example:
One-time Pad: secret s is string of length $|m|$.
$m = 1010101110101101$
$s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
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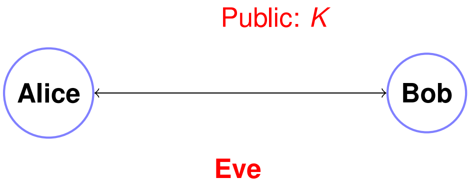

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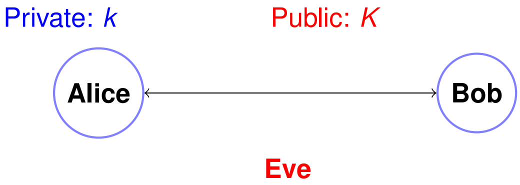

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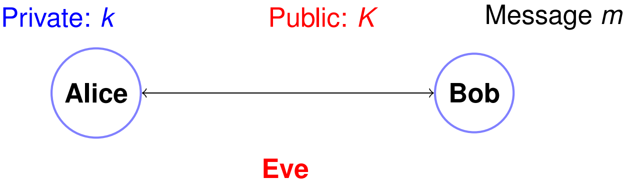

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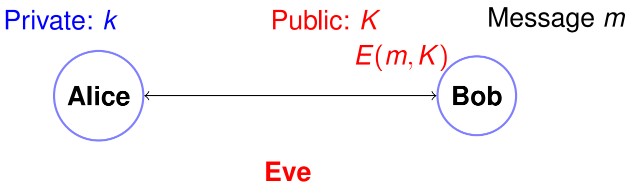

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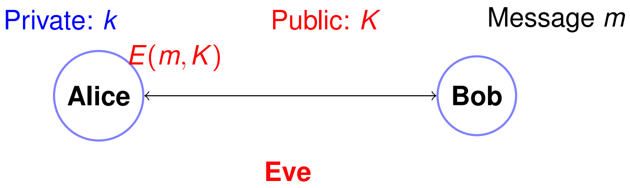

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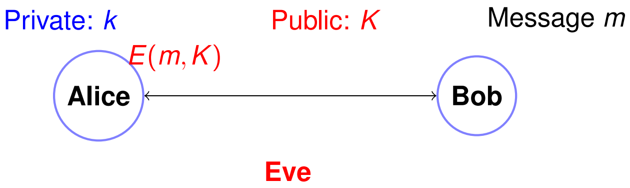

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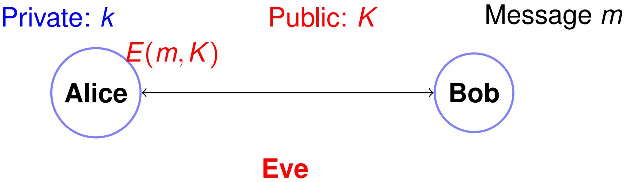

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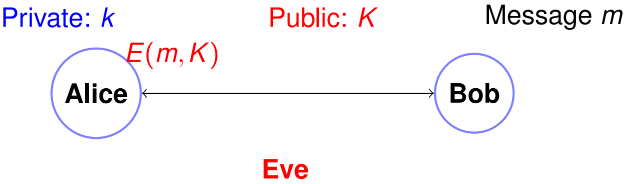

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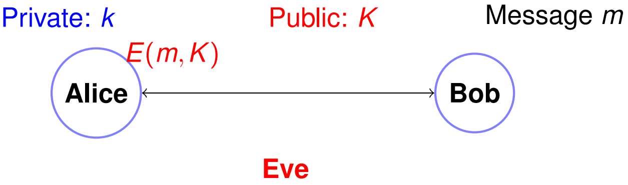

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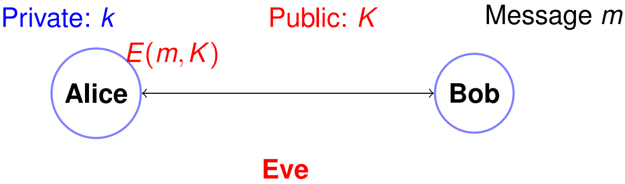

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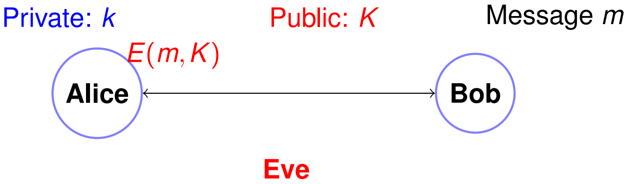

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Bob (and Eve and me and you and you ...) can encode.

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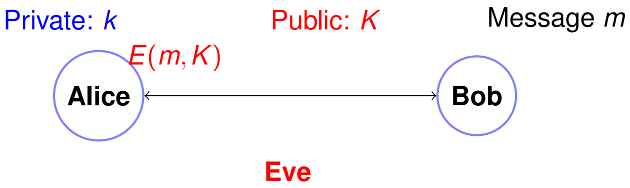

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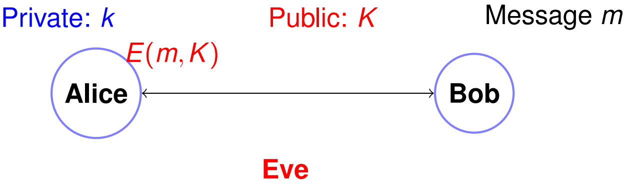

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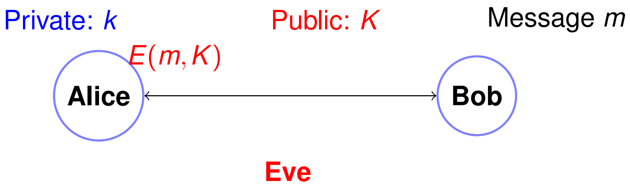

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Is this even possible?

Is public key crypto possible?

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$d = e^{-1} = -17 = 43 = \pmod{60}$

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In general, $O(N)$ or $O(2^n)$ multiplications!

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$51^{2} = (51)*(51) = 2601 \equiv 60 \pmod{77}$
$51^{4} = (51^{2})*(51^{2}) = 60*60 = 3600 \equiv 58 \pmod{77}$
$51^{8} = (51^{4})*(51^{4})$

Repeated squaring.

Notice: $43 = 32 + 8 + 2 + 1$ or 101011 in binary.
$51^{43} = 51^{32+8+2+1} = 51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} \pmod{77}$.
4 multiplications sort of...
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$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60) * (53) * (60) * (51) \equiv 2 \pmod{77}$.

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$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60)*(53)*(60)*(51) \equiv 2 \pmod{77}$.

Decoding got the message back!

Repeated squaring.

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Repeated Squaring took 9 multiplications

Repeated squaring.

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Decoding got the message back!

Repeated Squaring took 9 multiplications versus 43.

Repeated Squaring: x^y

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute x^1,

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2,$

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4,$

Repeated Squaring: x^y

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Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.
2. Multiply together x^i where the $(\log(i))$th bit of y (in binary) is 1.

Repeated Squaring: x^y

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   Example:

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Modular Exponentiation: $x^y \mod N$.

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

$O(n)$ multiplications.
$O(n^2)$ time per multiplication.

Repeated Squaring: x^y

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

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Conclusion: $x^y \mod N$

Repeated Squaring: x^y

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

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Conclusion: $x^y \mod N$ takes $O(n^3)$ time.

RSA is pretty fast.

Modular Exponentiation: $x^y \mod N$.

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$E(m,(N,e)) = m^e \pmod{N}$.

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versus $a^{k(p-1)(q-1)+1} = a \pmod{pq}$.

Similar, not same, but useful.

Correct decoding...

Fermat's Little Theorem: For prime p, and $a \not\equiv 0 \pmod{p}$,

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(B) $2^6 = 1 \mod 7$
(C) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ are distinct mod 7.
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From CRT: $y = x \pmod{p}$ and $y = x \pmod{q} \implies y = x$.

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All steps are polynomial in $O(\log N)$, the number of bits.

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Breaking in general sense $\implies$ factoring algorithm.

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CS161...

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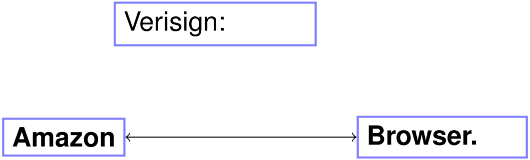

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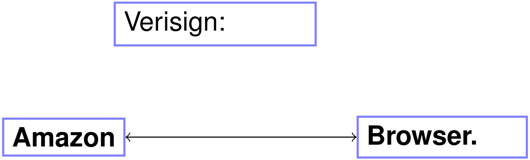

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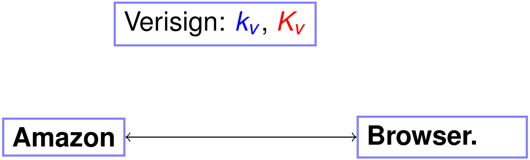

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Verisign's key: $K_V = (N, e)$ and $k_V = d$ ($N = pq$.)

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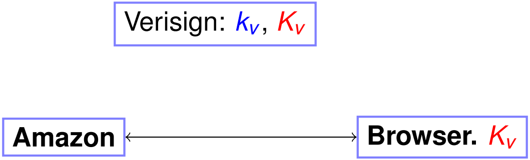

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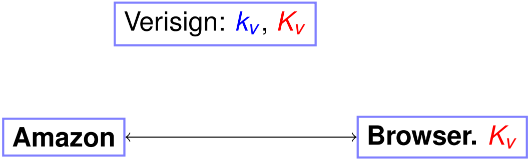

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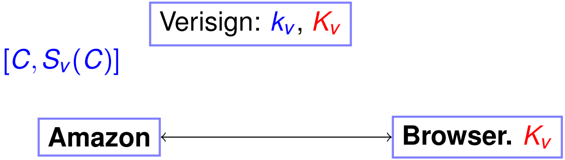

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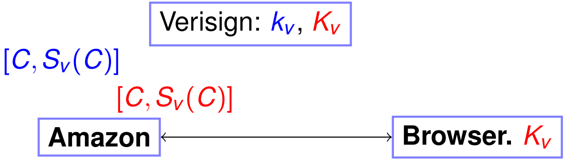

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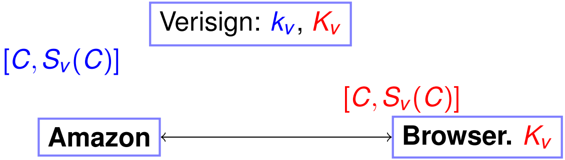

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$[C, S_v(C)]$ $\quad$ $C = E(S_V(C), k_V)$?

$[C, S_v(C)]$

Amazon ⟷ **Browser.** K_v

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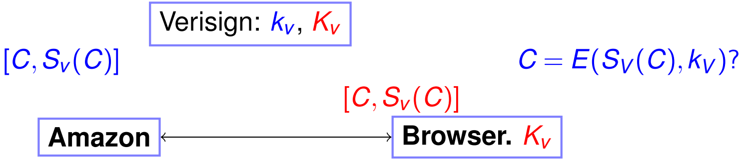

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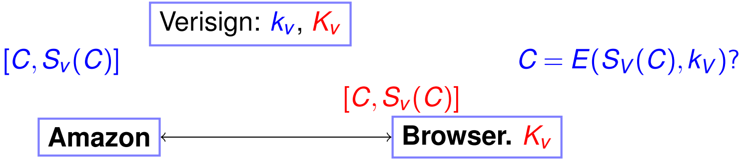

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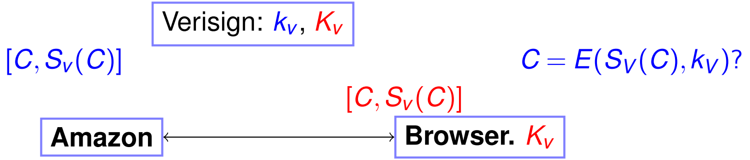

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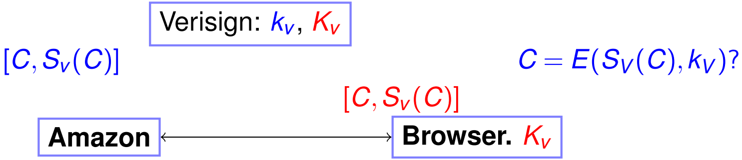

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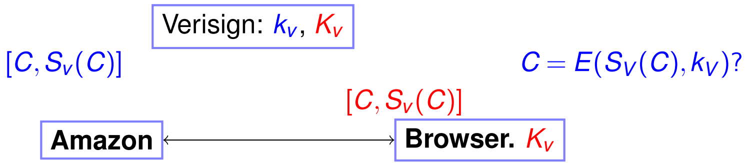

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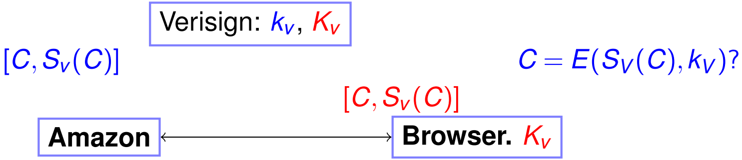

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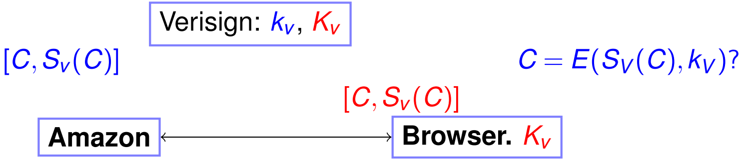

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Security: Eve can't forge unless she "breaks" RSA scheme.

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Public Key Cryptography:

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Signature authority has public key (N,e).

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RSA Scheme:

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