Lecture 5: Graphs.

Graphs!

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Graphs!

Definitions: model.

Lecture 5: Graphs.

Graphs!
- Definitions: model.
- Fact!

Lecture 5: Graphs.

Graphs!
- Definitions: model.
- Fact!

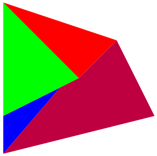
Lecture 5: Graphs.

Graphs!
  Definitions: model.
  Fact!
Planar graphs.

Lecture 5: Graphs.

Graphs!
  Definitions: model.
  Fact!
Planar graphs.
  Euler Again!!!!

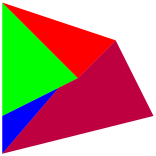
Map Coloring.

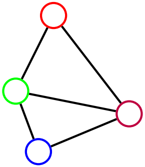


Map Coloring.

Map Coloring.

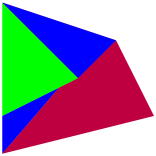
Map Coloring.

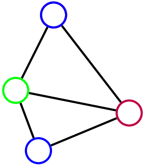




Fewer Colors?

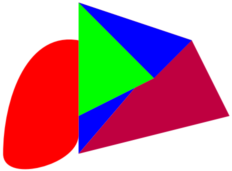
Map Coloring.

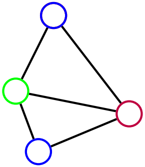




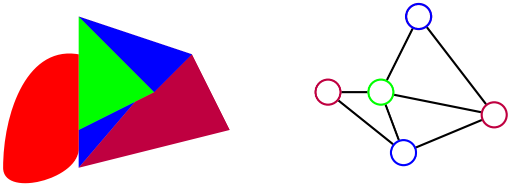
Yes! Three colors.

Map Coloring.

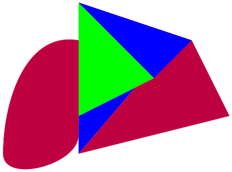


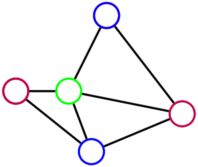


Map Coloring.

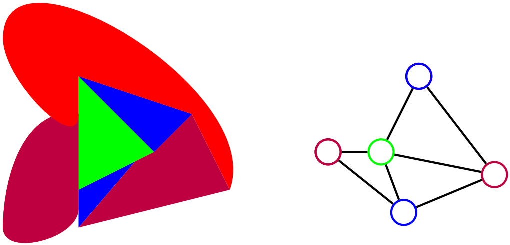


Map Coloring.





Map Coloring.



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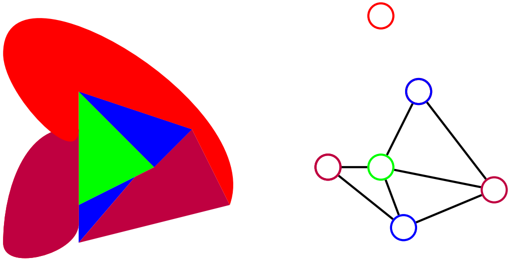
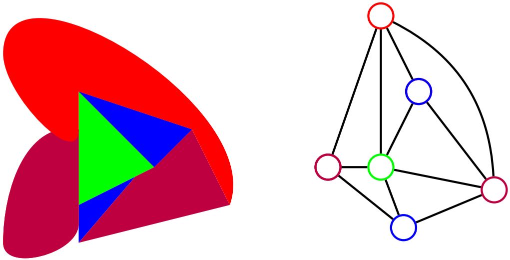
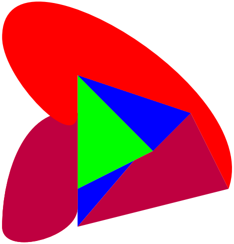
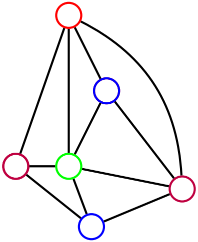


Map Coloring.



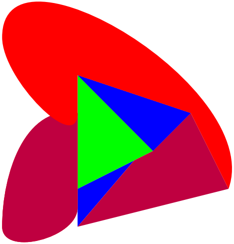
Map Coloring.

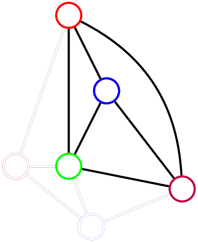




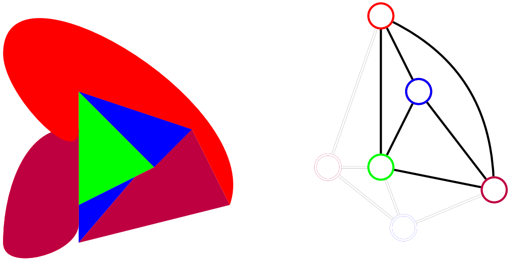
Fewer Colors?

Map Coloring.



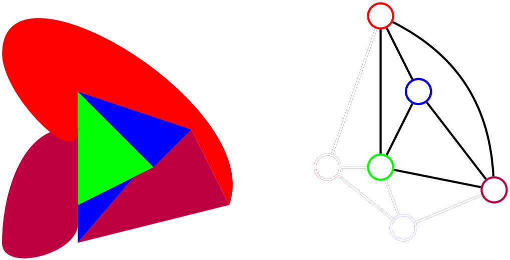


Map Coloring.



Four colors required!

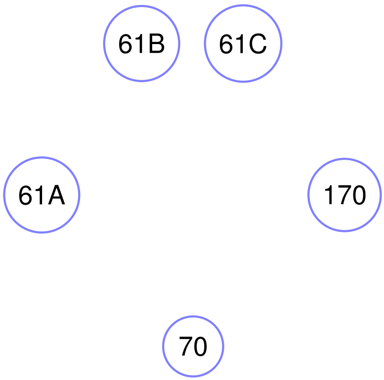
Map Coloring.



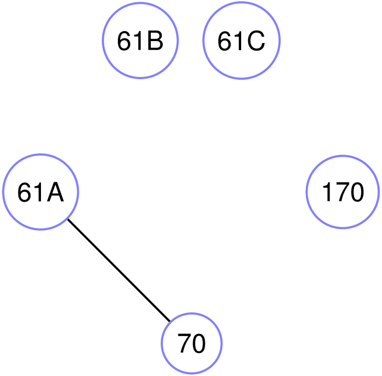
Four colors required!

Theorem: Four colors enough.

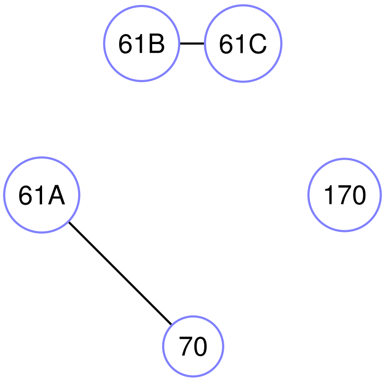
Scheduling: coloring.



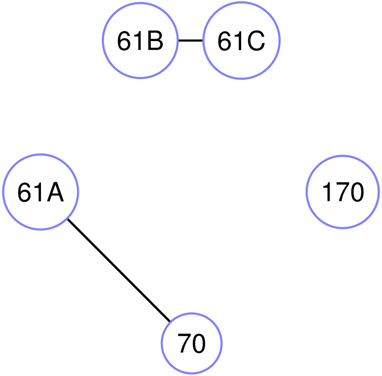
Scheduling: coloring.



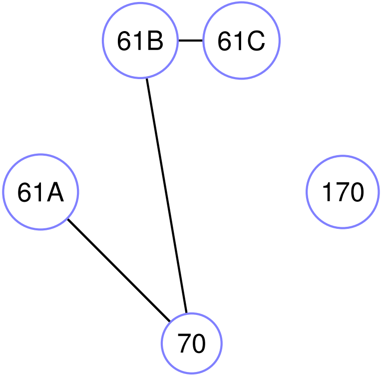
Scheduling: coloring.



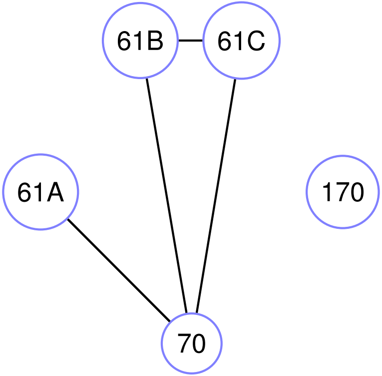
Scheduling: coloring.



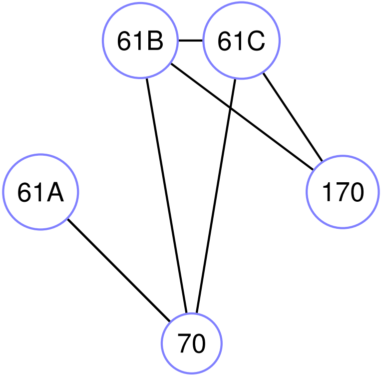
Scheduling: coloring.



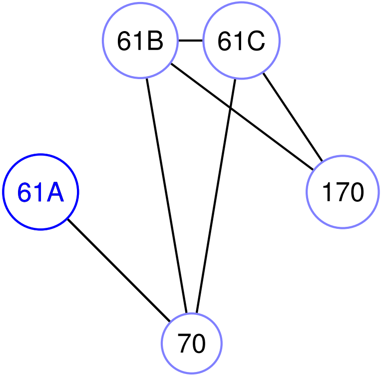
Scheduling: coloring.



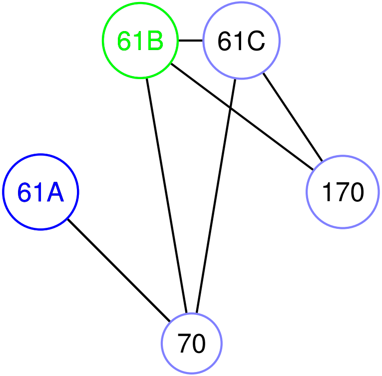
Scheduling: coloring.



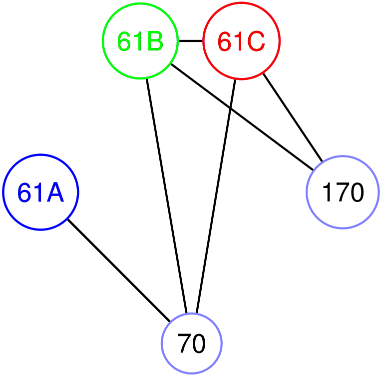
Scheduling: coloring.



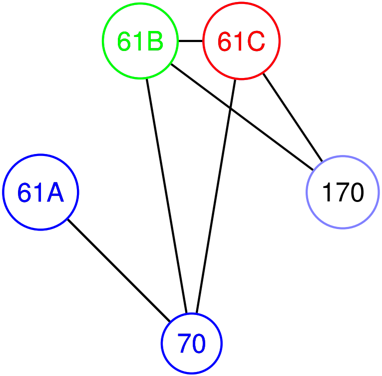
Scheduling: coloring.



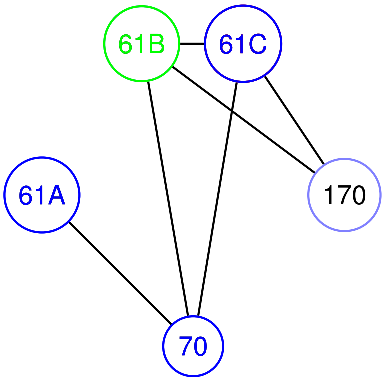
Scheduling: coloring.



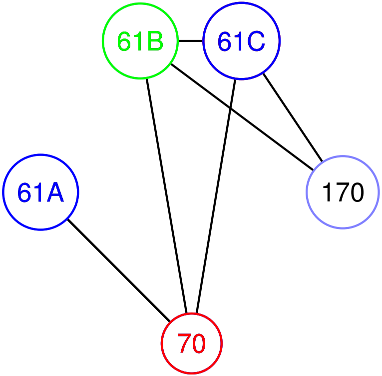
Scheduling: coloring.



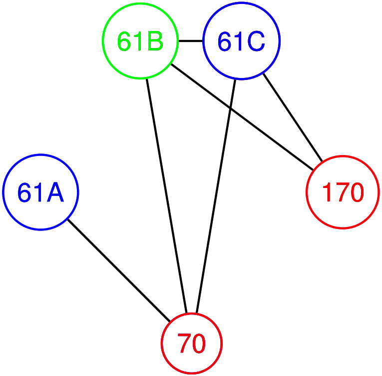
Scheduling: coloring.



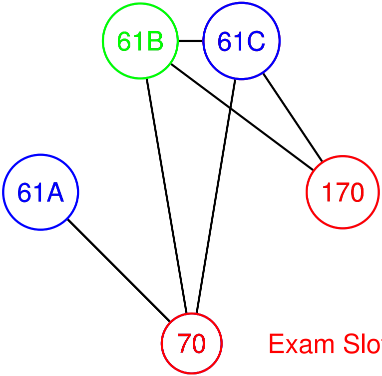
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

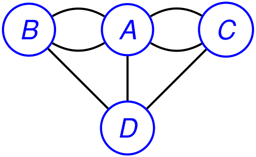


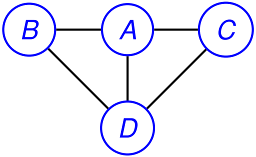
Exam Slot 1.

Exam Slot 2.

Exam Slot 3.

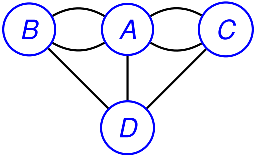
Graphs: formally.

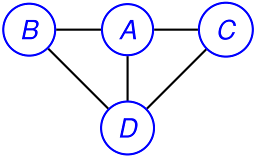




Graph:

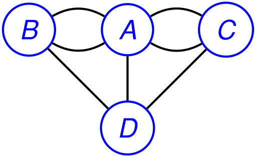
Graphs: formally.

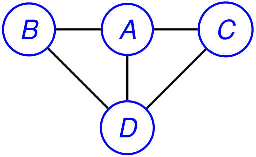




Graph: $G = (V, E)$.

Graphs: formally.

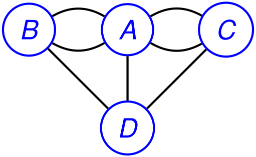


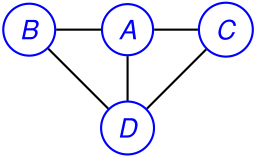


Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.



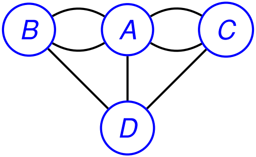


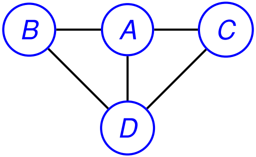
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.





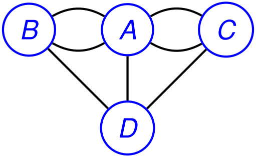
Graph: $G = (V, E)$.

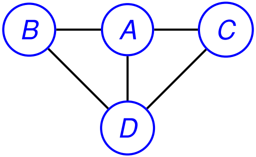
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.





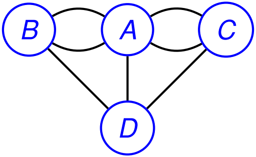
Graph: $G = (V, E)$.

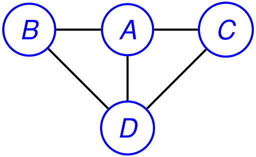
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.





Graph: $G = (V, E)$.

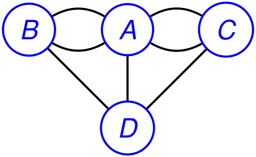
V - set of vertices.

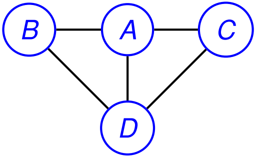
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.





Graph: $G = (V, E)$.

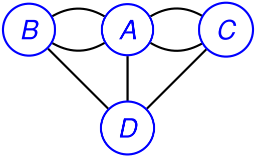
V - set of vertices.

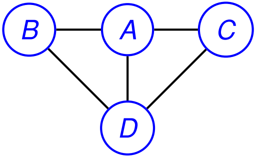
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.





Graph: $G = (V, E)$.

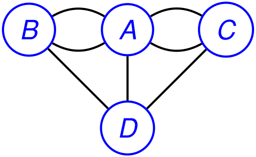
V - set of vertices.

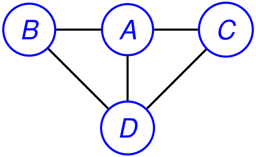
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.





Graph: $G = (V, E)$.

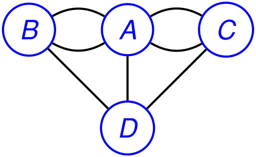
V - set of vertices.

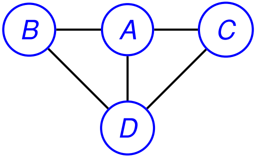
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

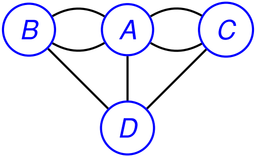
$\{A, B, C, D\}$

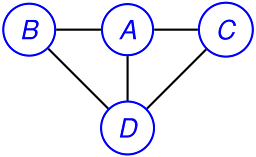
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

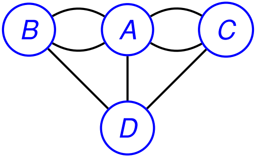
$E \subseteq V \times V$ - set of edges.

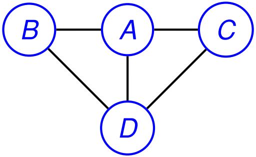
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

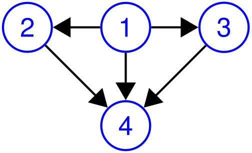
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

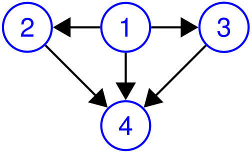
Multigraph above.

Directed Graphs



$G = (V, E)$.

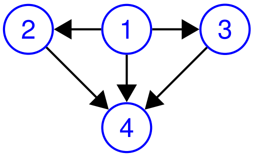
Directed Graphs



$G = (V, E)$.

V - set of vertices.

Directed Graphs

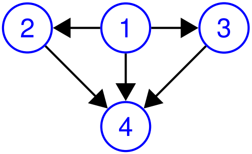


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



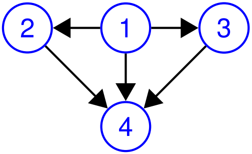
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

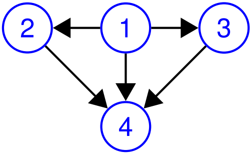
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

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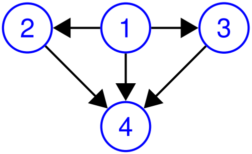
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3),$

Directed Graphs



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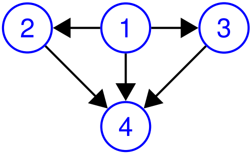

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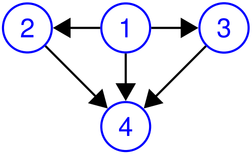

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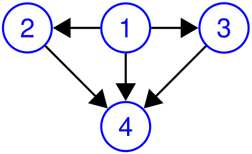
One way streets.

Directed Graphs



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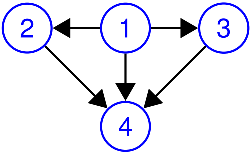
One way streets.
Tournament:

Directed Graphs



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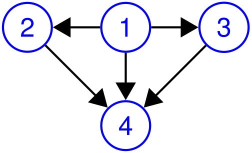
One way streets.
Tournament: 1 beats 2,

Directed Graphs



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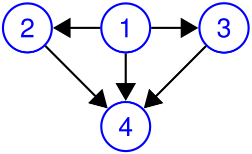
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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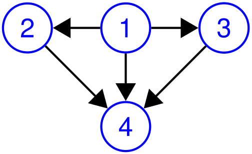
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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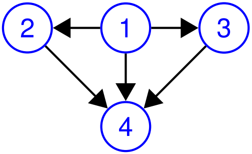
Directed Graphs



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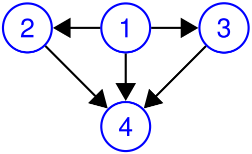
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



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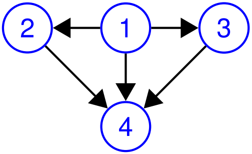
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



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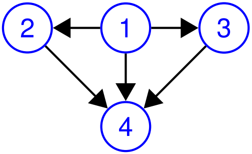
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs



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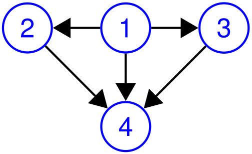
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs



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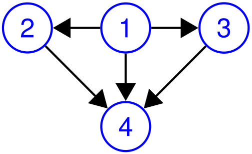
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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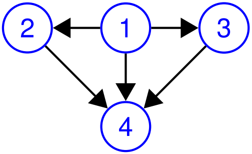
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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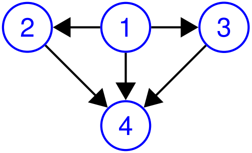
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

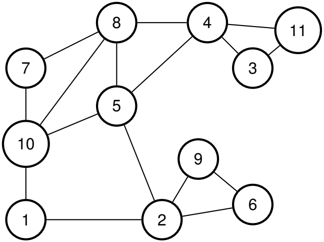
Graph: $G = (V, E)$

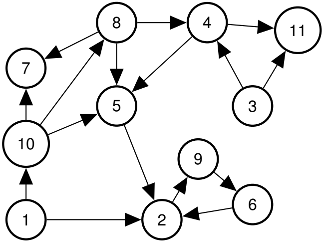
neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



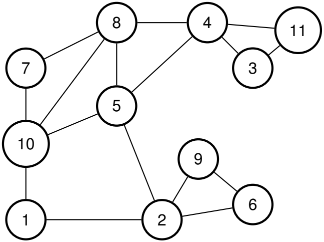


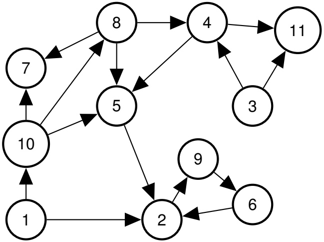
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



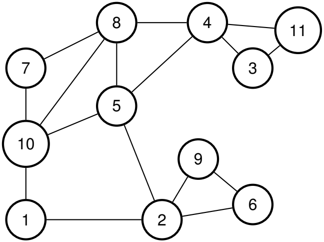


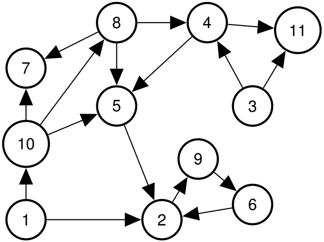
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



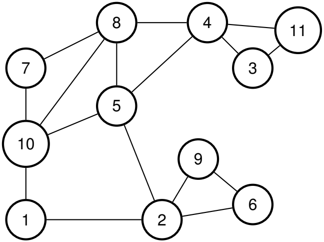


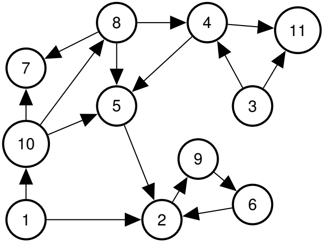
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



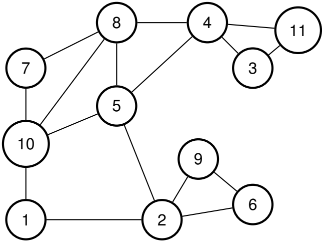


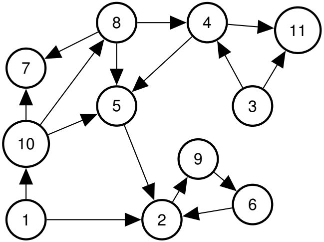
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



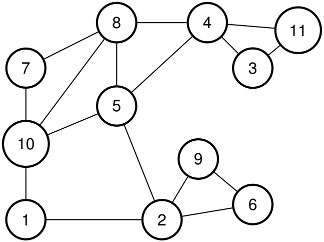


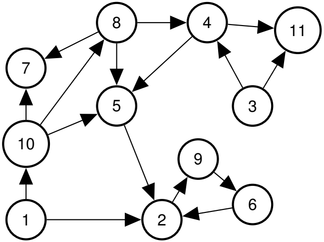
Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





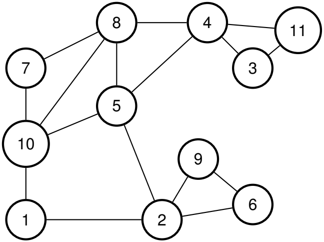
Neighbors of 10? 1,5,7, 8.

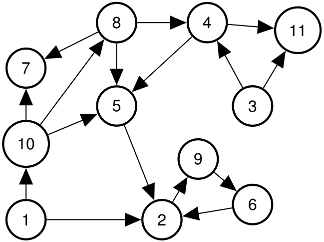
u is neighbor of v if $\{u, v\} \in E$.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Edge $\{10, 5\}$ is incident to

Graph Concepts and Definitions.

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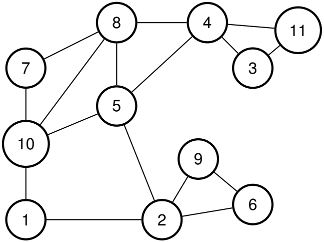
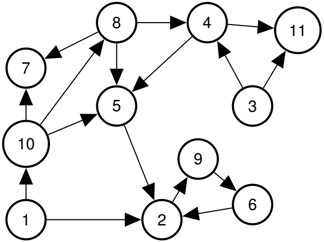
neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

u is neighbor of v if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

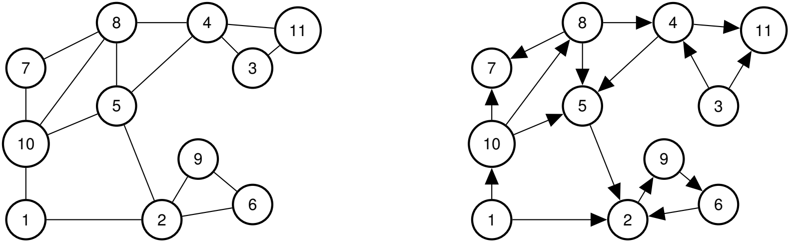
Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Degree of vertex 1? 2

Graph Concepts and Definitions.

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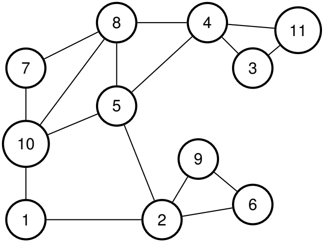

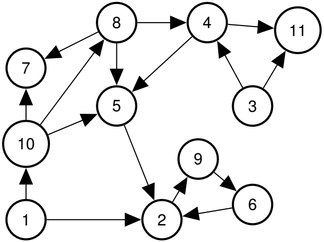

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Edge $\{u, v\}$ is incident to u and v.

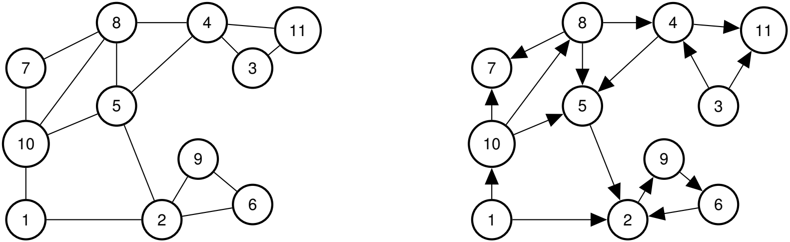
Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Equals number of neighbors in simple graph.

Graph Concepts and Definitions.

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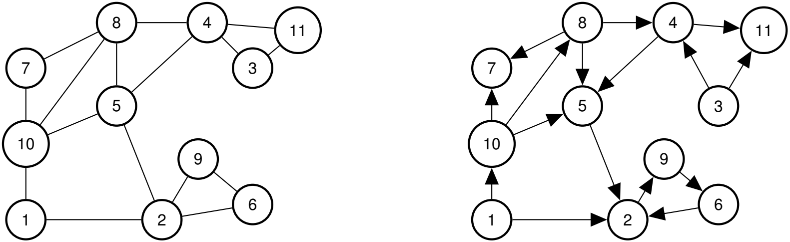

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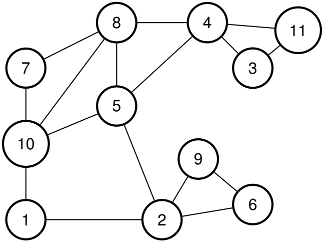

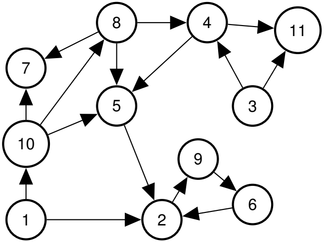

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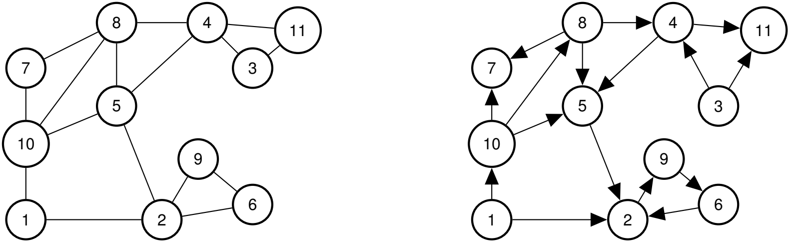
Equals number of neighbors in simple graph.

Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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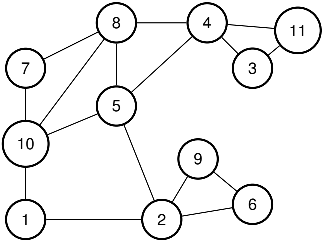
Directed graph?

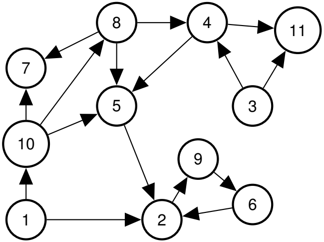
In-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Equals number of neighbors in simple graph.

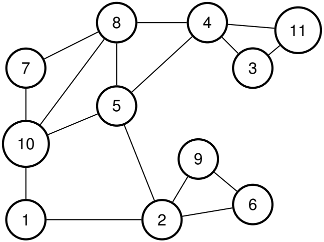
Directed graph?

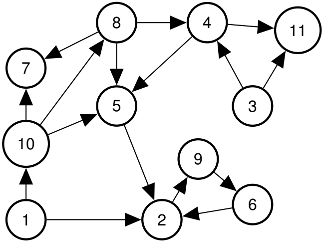
In-degree of 10? 1

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Equals number of neighbors in simple graph.

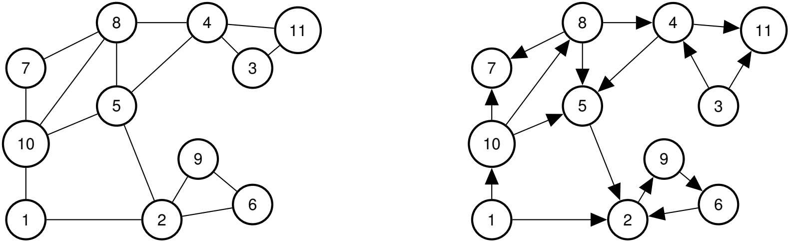
Directed graph?

In-degree of 10? 1   Out-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



Neighbors of 10? 1,5,7, 8.

u is neighbor of v if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Equals number of neighbors in simple graph.

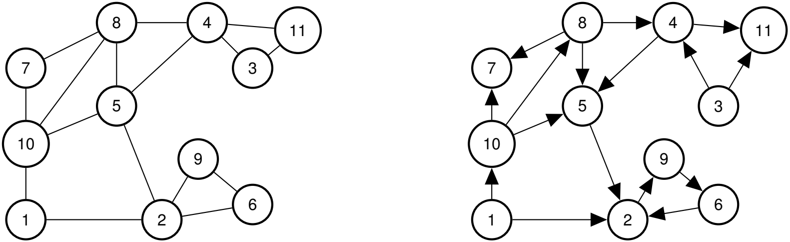
Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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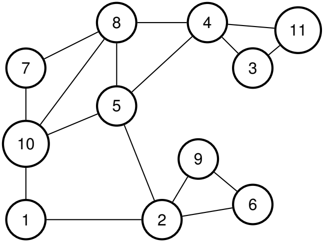

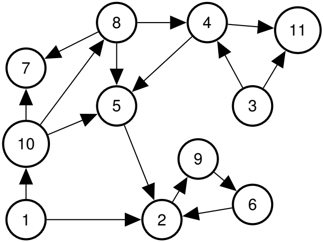

Edge $(8, 5)$ **is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

Graph Concepts and Definitions.

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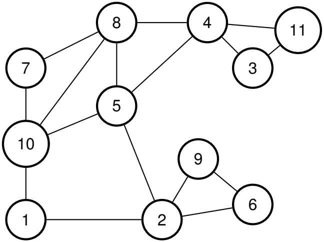

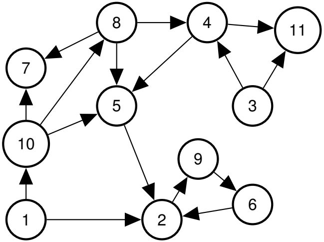

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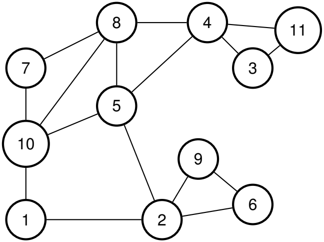

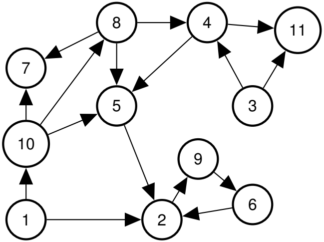

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The degree of a vertex is:

(A) The number of edges incident to it.
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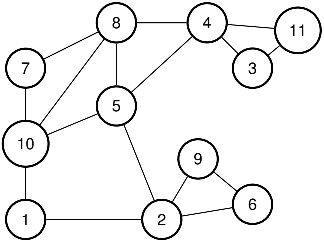

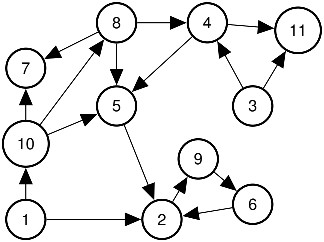

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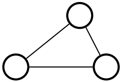
Not (A)!



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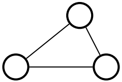
Not (A)! Triangle.



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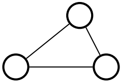

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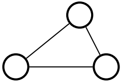

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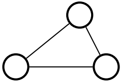

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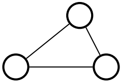

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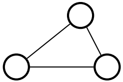

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What? For triangle number of edges is 3, the sum of degrees is 6.

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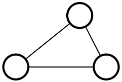

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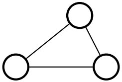

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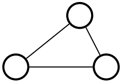

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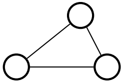

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Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Recall:

Quick Proof of an Equality.

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edge, (u,v), is incident to endpoints, u and v.

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degree of u number of edges incident to u

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What is degree v? Incidences corresponding to v!

Quick Proof of an Equality.

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

Poll: Proof of “handshake” lemma.

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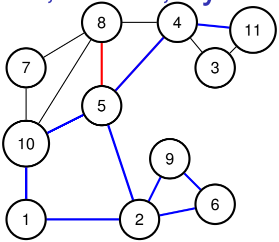
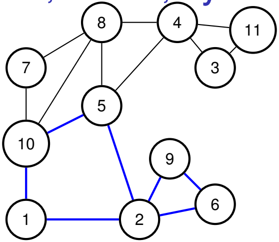
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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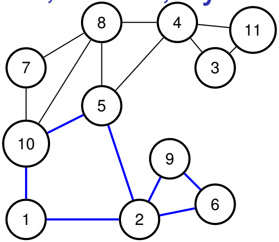
Path?

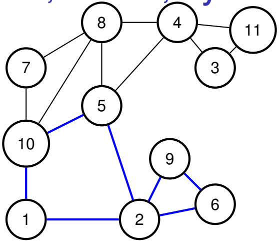
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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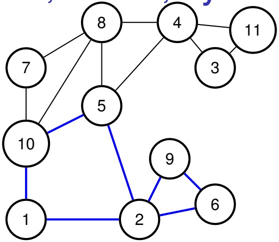

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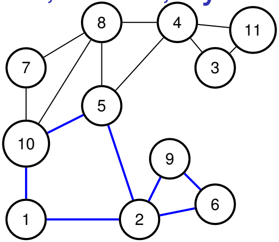

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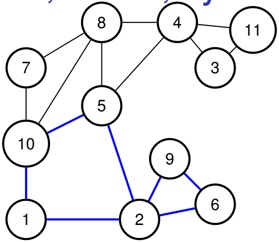
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$?

Paths, walks, cycles, tour.



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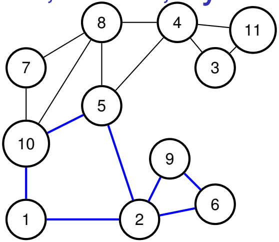

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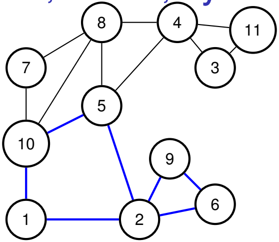
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

Paths, walks, cycles, tour.



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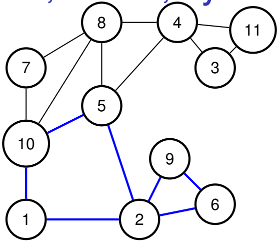
Quick Check!

Paths, walks, cycles, tour.



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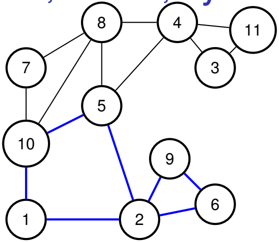
Quick Check! Length of path?

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

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Quick Check! Length of path? k vertices

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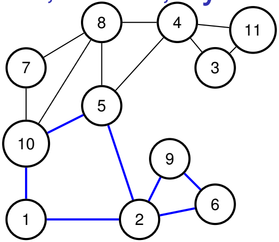

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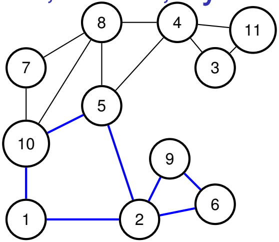

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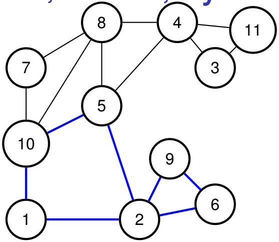

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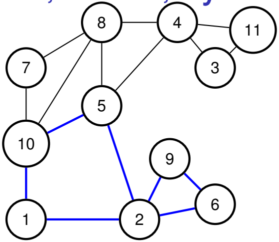

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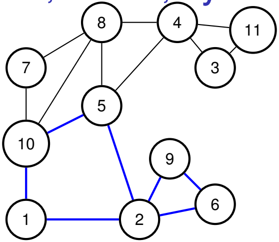

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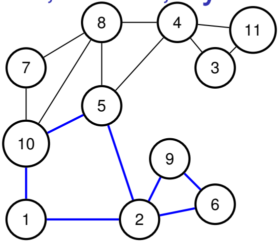

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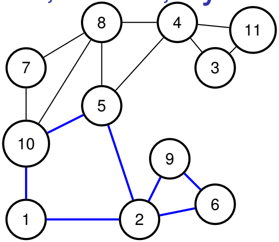

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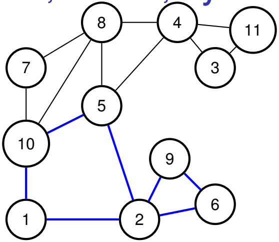

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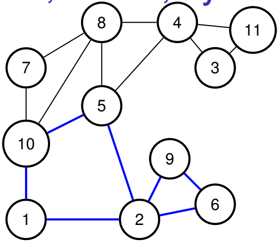

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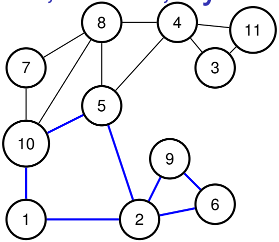

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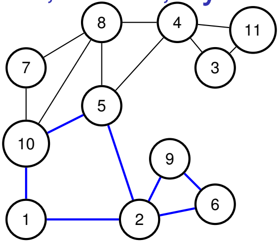

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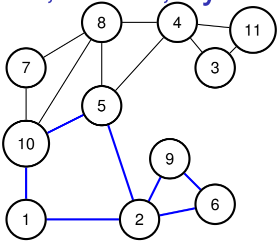

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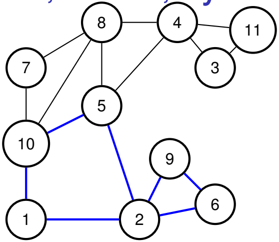

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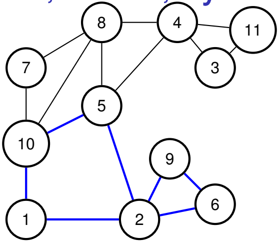

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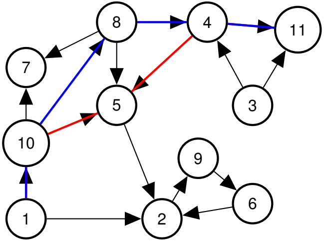

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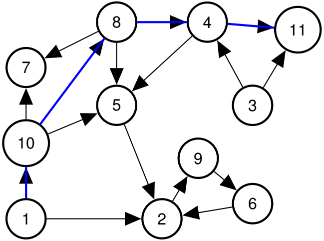

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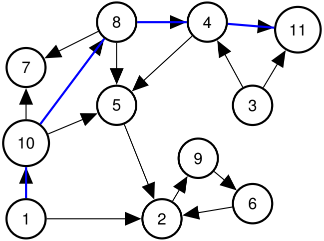

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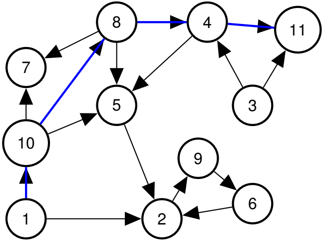

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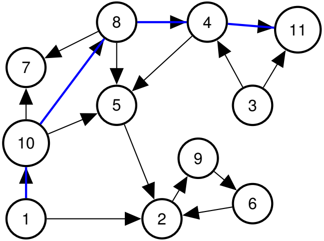

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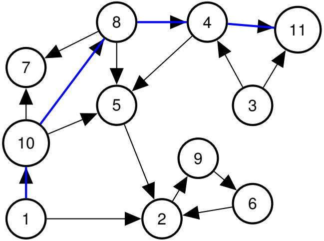

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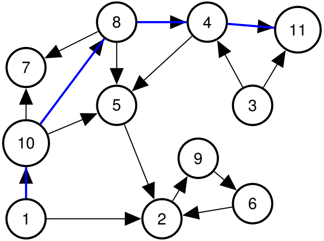

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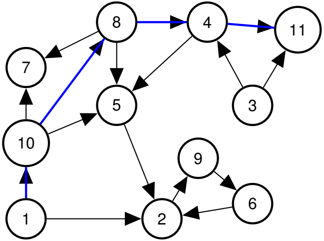

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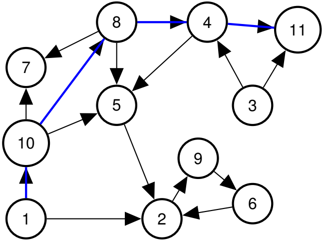

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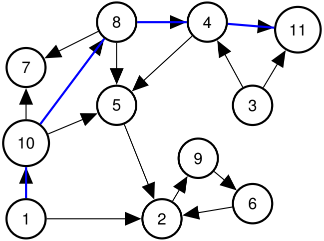

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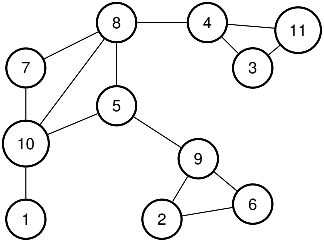
Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

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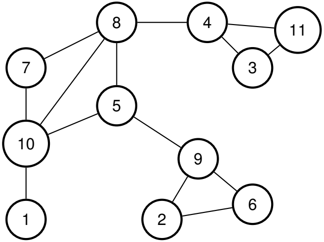

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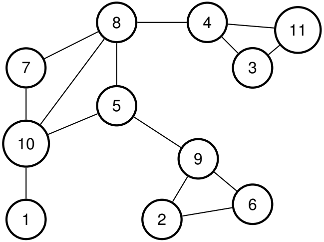

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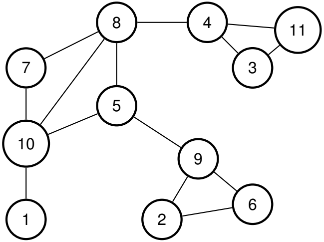

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Is graph connected?

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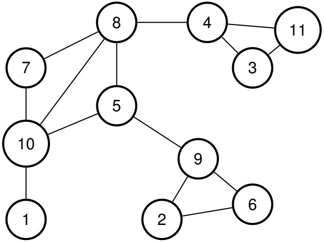

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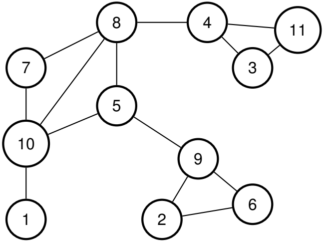

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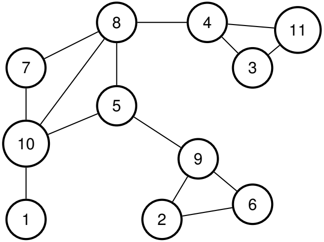

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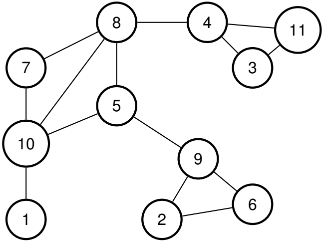

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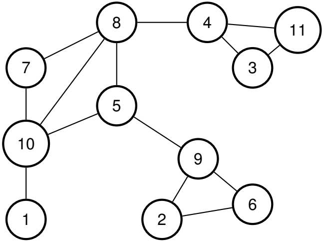

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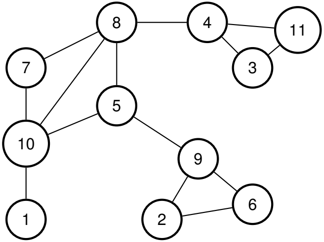

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May not be simple!

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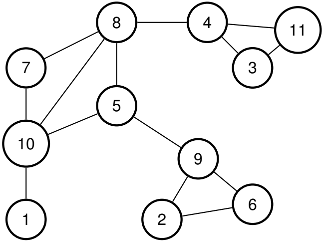

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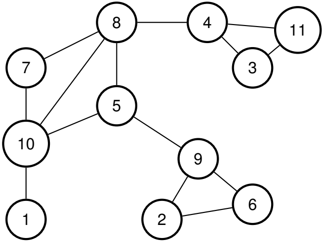

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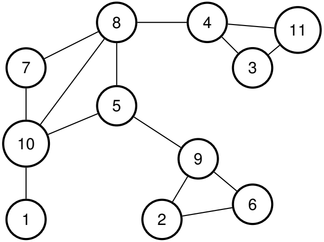

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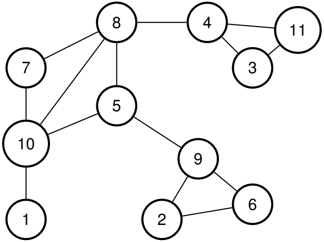
Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

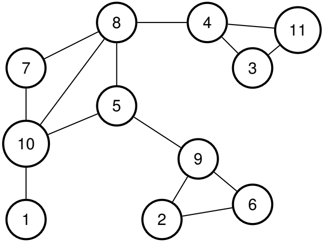
A connected graph is a graph where all pairs of vertices are connected.

If one vertex x is connected to every other vertex.
Is graph connected? Yes? No?

Proof: Use path from u to x and then from x to v. $\square$

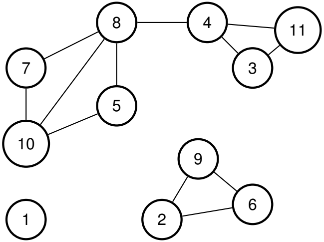
May not be simple!
Either modify definition to walk.
Or cut out cycles. .

Connected Components: Quiz.



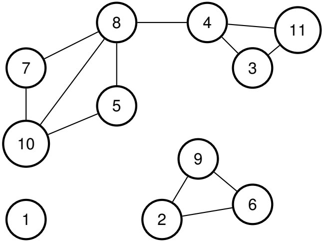
Is graph above connected?

Connected Components: Quiz.



Is graph above connected? Yes!

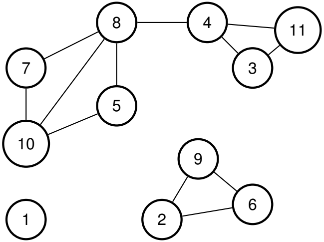
Connected Components: Quiz.



Is graph above connected? Yes!

How about now?

Connected Components: Quiz.



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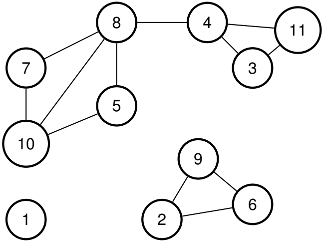

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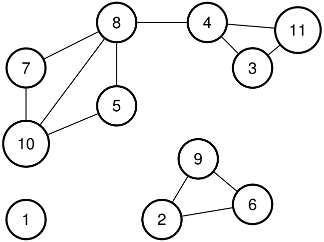
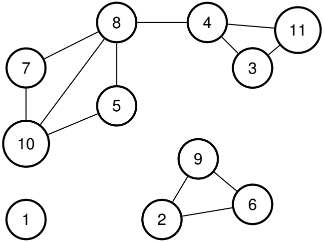


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components: Quiz.



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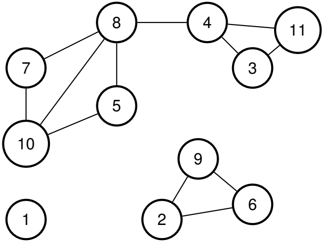
Connected component - maximal set of connected vertices.

Connected Components: Quiz.



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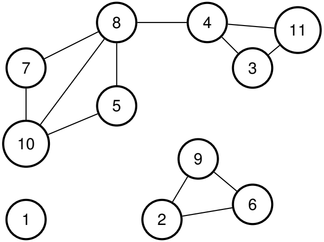
Connected component - maximal set of connected vertices.

Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected Components: Quiz.



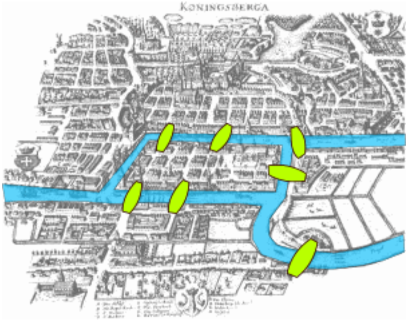
Is graph above connected? Yes!

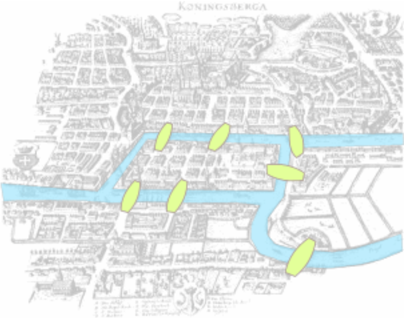
How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.
Connected component - maximal set of connected vertices.
Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?





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Yes? No?
We will see!

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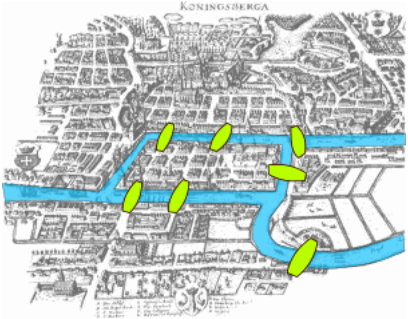

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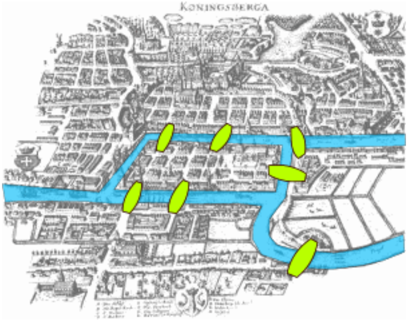

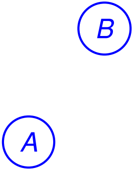

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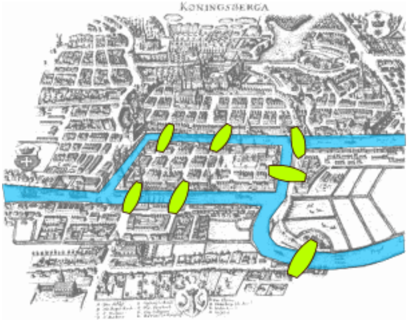

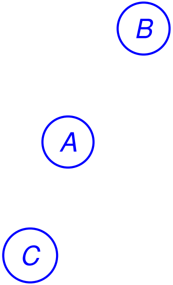

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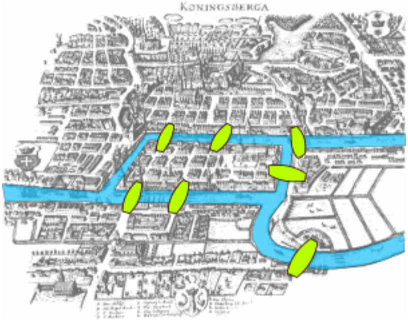

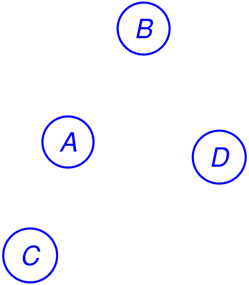

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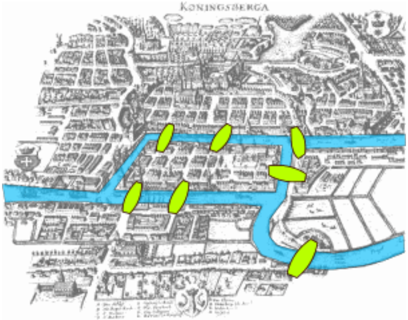

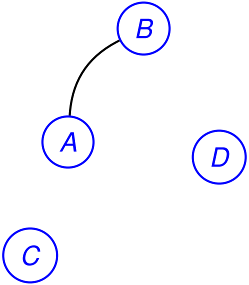

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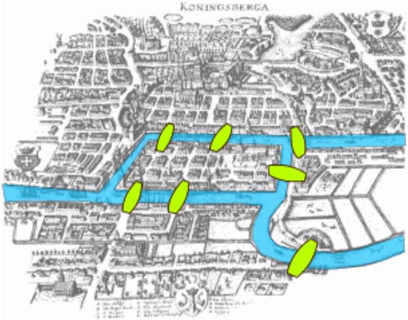

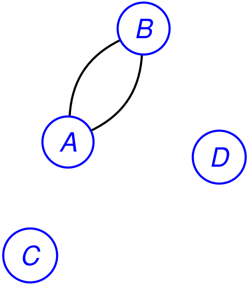

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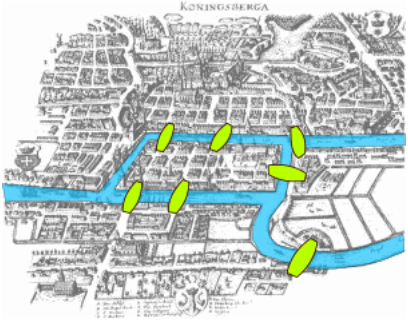

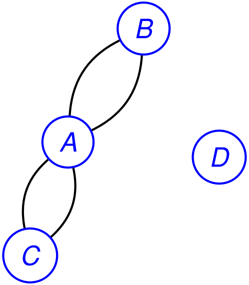

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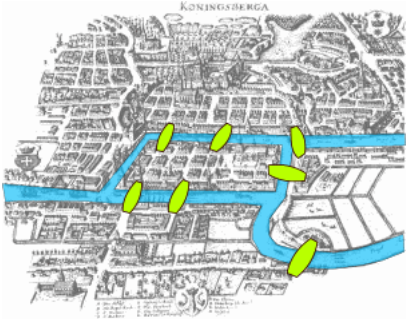

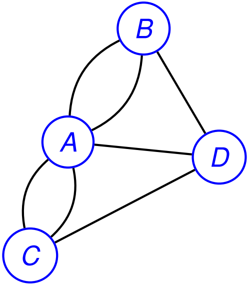

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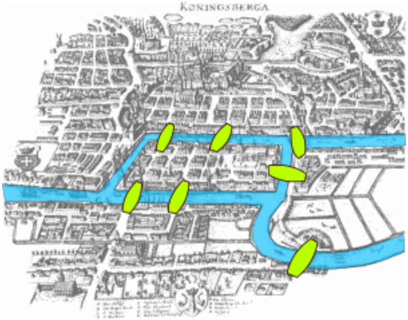

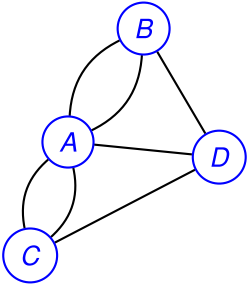

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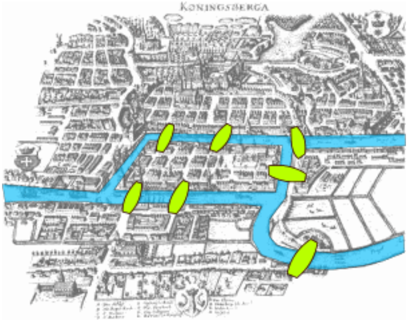

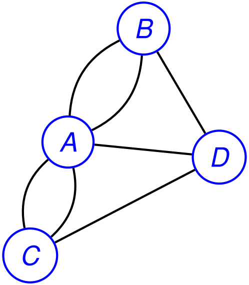

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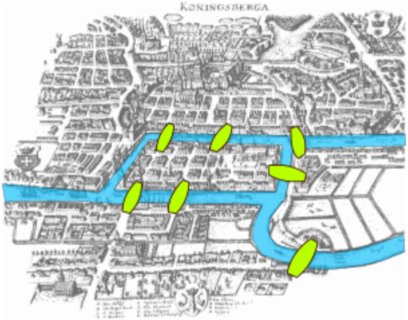

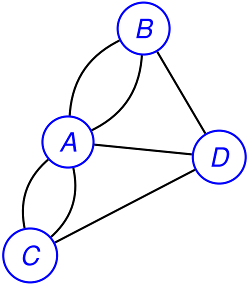

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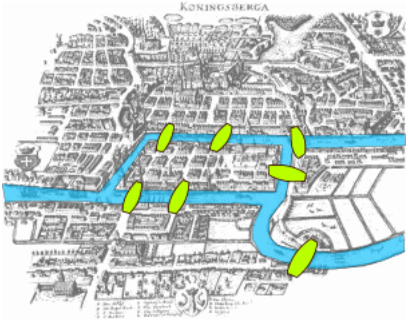

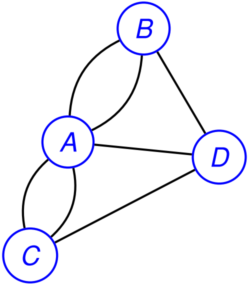

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Yes?   No?
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Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

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Eulerian Tour is connected so graph is connected.

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Tour enters and leaves vertex *v* on each visit.

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Uses two incident edges per visit.

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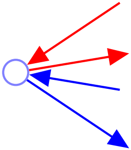

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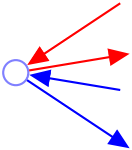

When you enter,

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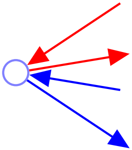

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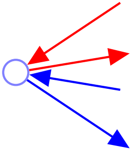

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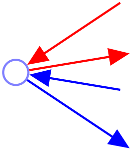

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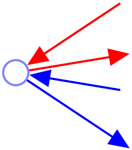

When you enter, you can leave.
For starting node,

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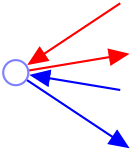

When you enter, you can leave.
For starting node, tour leaves first

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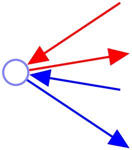

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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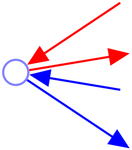

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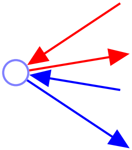

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
We will give an algorithm.

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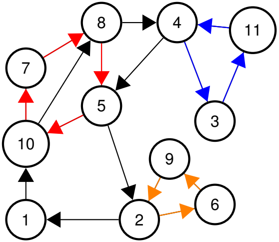

1. Take a walk starting from v (1) on "unused" edges

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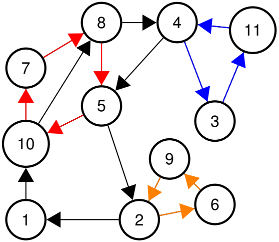

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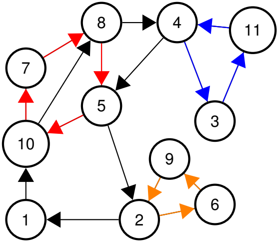

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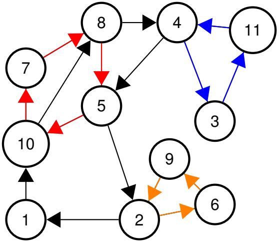

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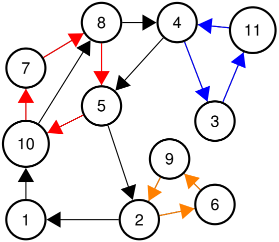

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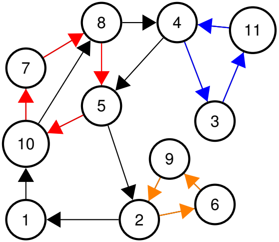

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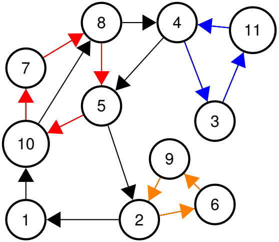

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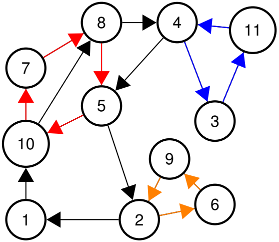

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2. Remove tour, C.

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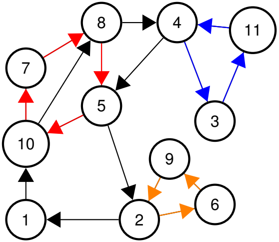

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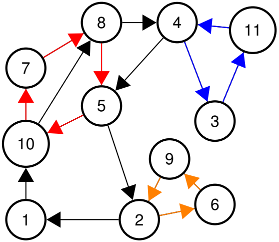

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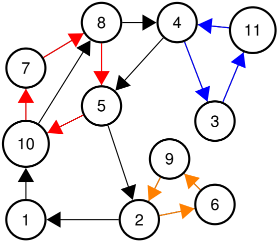

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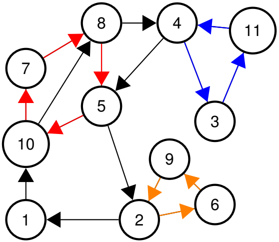

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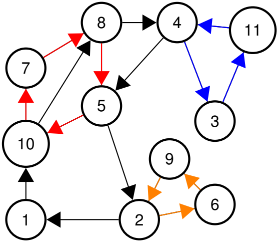

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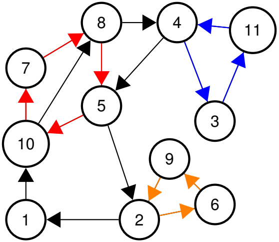

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    Example: $v_1 = 1$,

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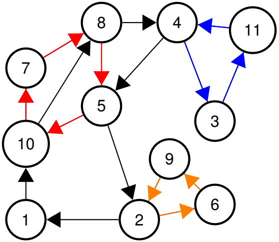

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    Example: $v_1 = 1$, $v_2 = 10$,

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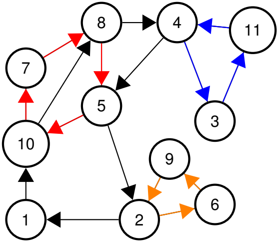

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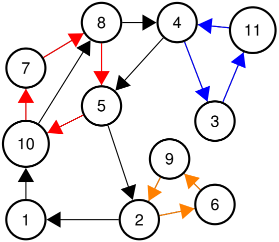

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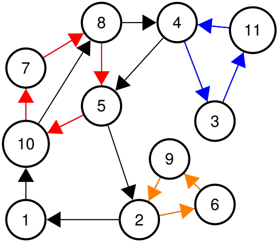

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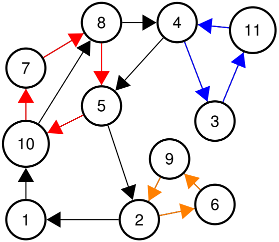

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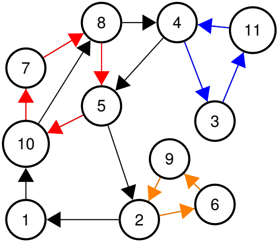

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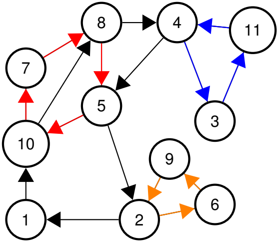

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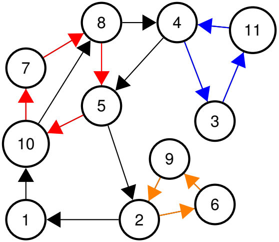

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   1,10,7,8,5,10

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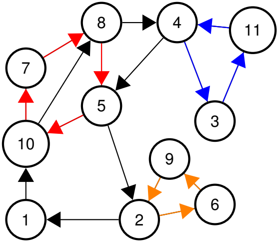

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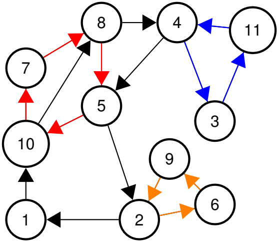

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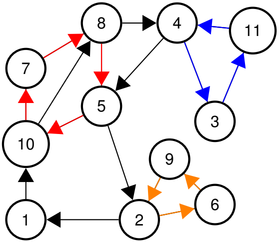

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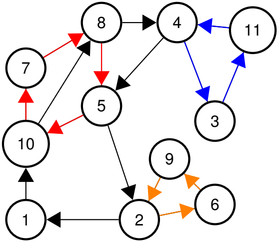

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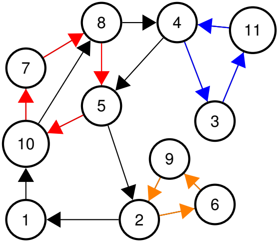

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Proof of Claim: Even degree.

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Proof of Claim: Even degree. If enter, can leave

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Proof of Claim: Even degree. If enter, can leave except for v.

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2. Remove cycle, C, from G.

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2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)

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Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

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Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.

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Let components be $G_1, \ldots, G_k$.
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Why is there a v_i in C?

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G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
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(A) Removing a tour leaves a graph of even degree.
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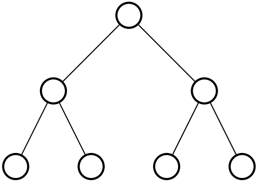
Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

Trees.

Definitions:

Trees.

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A connected graph without a cycle.

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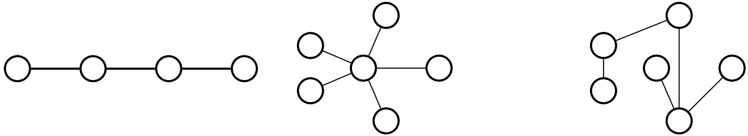

no cycle and connected?

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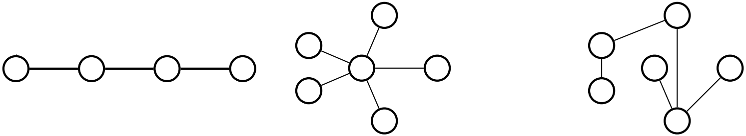

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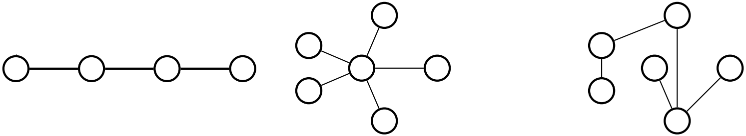

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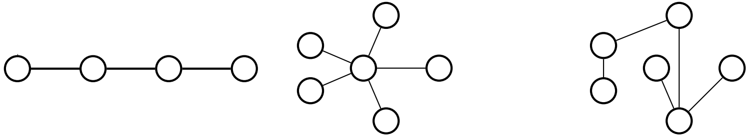

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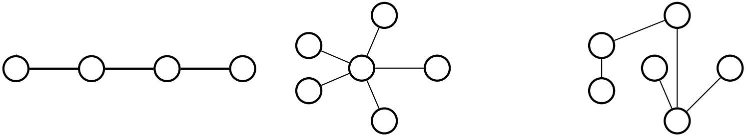

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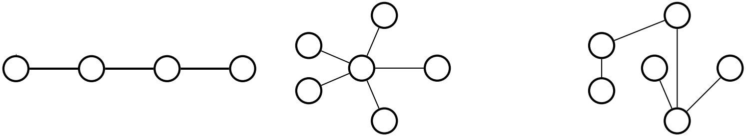

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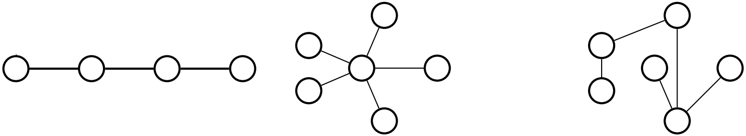

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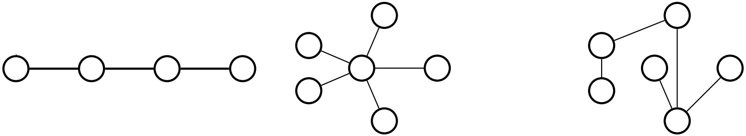

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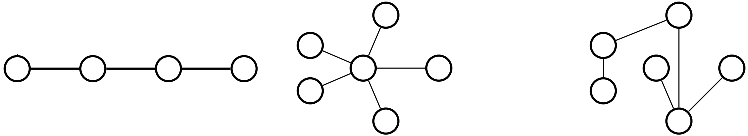

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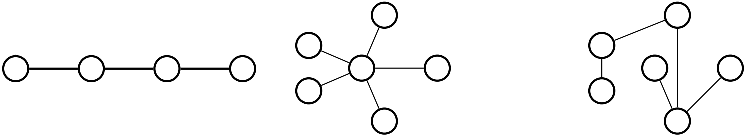

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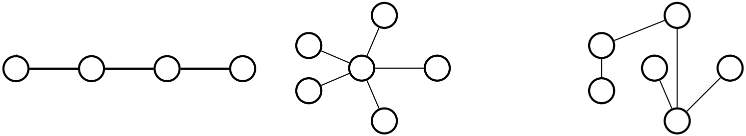

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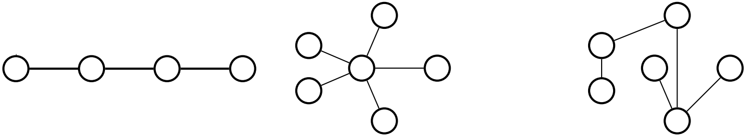

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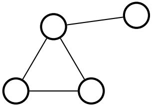

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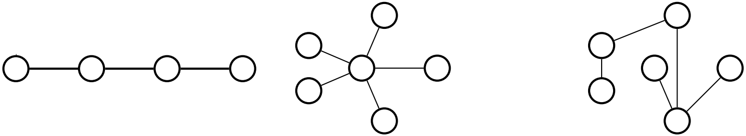

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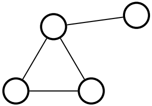

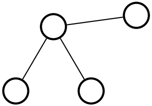

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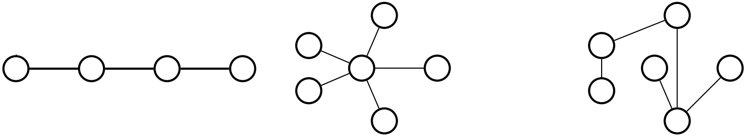

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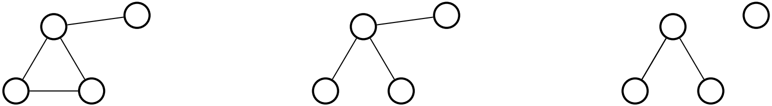

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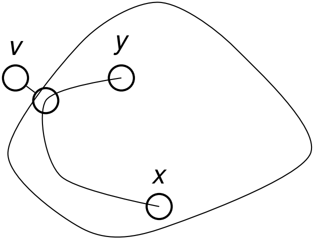

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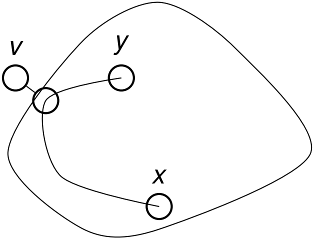

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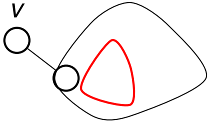
Proof of $\implies$:



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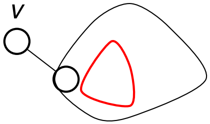

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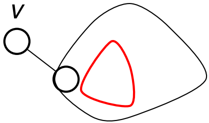

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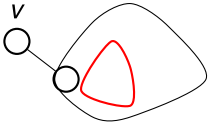

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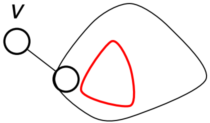

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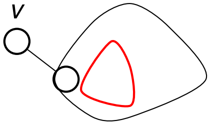
"G connected and has $|V|-1$ edges" $\Longrightarrow$
"G is connected and has no cycles."

Proof of $\Longrightarrow$: By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:

Proof of only if.



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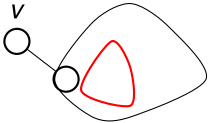

Thm:
"G connected and has $|V| - 1$ edges" $\implies$
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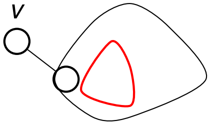
Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.

Proof of only if.



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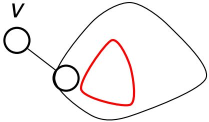
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|E|=2(|V|-1)=2|V|-2$

Proof of only if.



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Induction Step:
Claim: There is a degree 1 node.
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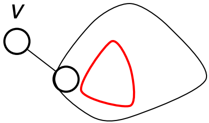
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $2 - 2/|V|$

Proof of only if.



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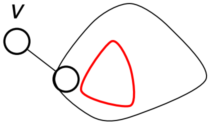
Not everyone is bigger than average!

Proof of only if.



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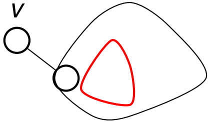

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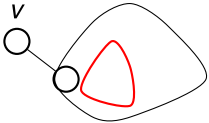
By degree 1 removal lemma, $G-v$ is connected.

Proof of only if.



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$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction

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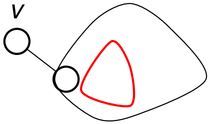

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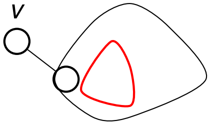

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And no cycle in G since degree 1 cannot participate in cycle.

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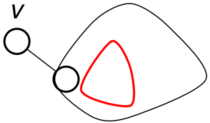

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Proof:
Walk from a vertex using untraversed edges.

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Can't visit more than once since no cycle.

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Entered.

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Can't visit more than once since no cycle.
Entered. Didn't leave.

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G has one more or $|V|-1$ edges.

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Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

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(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

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(B), (C), (D) are true

Lecture in a minute.

Graphs.

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Graphs.
 Basics.

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Algorithm for Eulerian Tour.

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Trees: degree 1 lemma $\implies$ several definitions.

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Planar Graphs: intro.