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CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Divides.

$a|b$ means

(A) There exists $k \in \mathbb{Z}$, with $a = kb$.

(B) There exists $k \in \mathbb{Z}$, with $b = ka$.

(C) There exists $k \in \mathbb{N}$, with $b = ka$.

(D) There exists $k \in \mathbb{Z}$, with $k = ab$.

(E) a divides b

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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Goal: $P \implies Q$
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...
Therefore Q.

Another direct proof.

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Add $99a + 11b$ to both sides.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Add $99a + 11b$ to both sides.

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Let D_3 be the 3 digit natural numbers.

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Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

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□

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Did we prove?

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- Proof assumed no primes *in between* p_k and q.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

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The fourth case is the only one possible, so the lemma follows. □

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

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Question: Which case holds?

Proof by cases.

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One of the cases is true so theorem holds. $\square$

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

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Proof: Assume $3 = 4$.

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Start with $12 = 12$.

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Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$

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Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y)\end{aligned}$$

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(A) Assumed what you were proving.

(B) No problem. Its fine.

(C) $x - y$ is zero.

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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!
Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

Poll. What's the biggest number?

(A) 100

(B) 101

(C) n+1

(D) infinity.

(E) This is about the "recursive leap of faith."