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Modular Arithmetic Fact: Exactly 1 polynomial of degree $\leq d$ with arithmetic modulo prime p contains $d+1$ pts.

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Uniqueness Fact. At most one degree d polynomial hits $d+1$ points.

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$R(x) = Q(x) - P(x)$ has $d+1$ roots and is degree d.

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In general, divide $P(x)$ by $(x-a)$ gives $Q(x)$ and remainder r.

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r is degree 0 polynomial..or a constant!

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  At most d roots for degree d polynomial.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

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Roubustness: Any k knows secret.

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Faster versions in practice are almost as efficient.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

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3 kids hand out 3 points. Any two know the line.

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Infinite number for reals, rationals, complex numbers!

Secret Sharing.

n people, k is enough.

(A) The modulus needs to be at least $n+1$.
(B) The modulus needs to be at least k.
(C) Use degree k polynomial, hand out n points.
(D) Use degree n polynomial, hand out k points.
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(G) The modulus needs to be at least 2^s, where s is size of secret.

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(A), (B), (E), (F)

Erasure Codes.

Satellite

GPS device

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3 packet message.

GPS device

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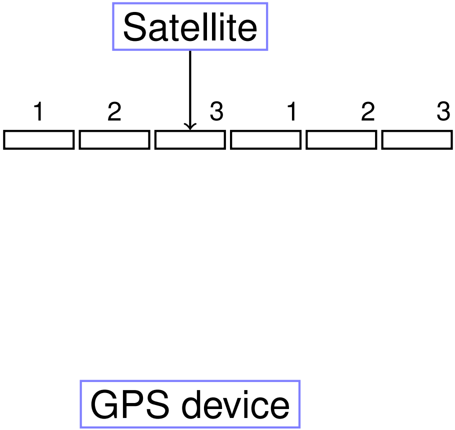
3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



3 packet message. So send 6!

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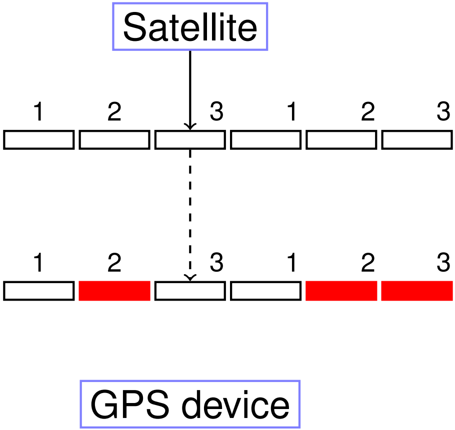

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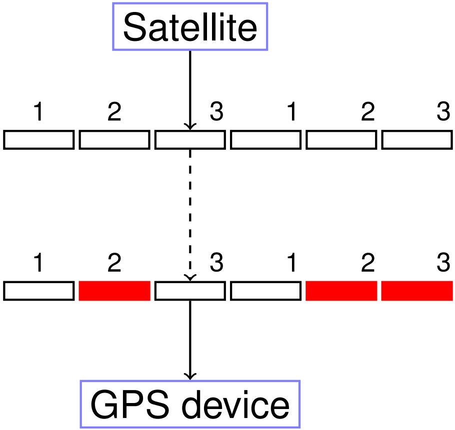

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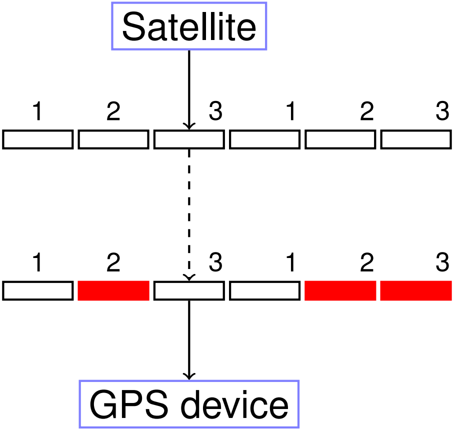

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Gets packets 1,1,and 3.

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n packet message, channel that loses k packets.

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Any n packets should allow reconstruction of n packet message.

Any n point values allow reconstruction of degree $n-1$ polynomial.

Alright!!!!!!

Use polynomials.

The Scheme

Problem: Want to send a message with n packets.

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Erasure Codes.

Satellite

GPS device

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Satellite

n packet message.

GPS device

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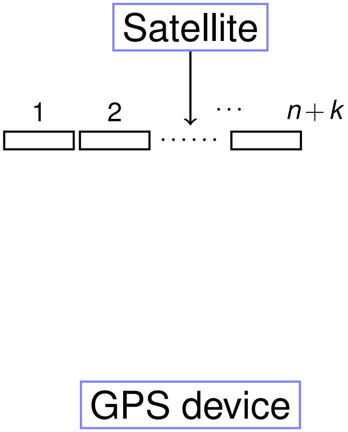
Lose *k* packets.

GPS device

Erasure Codes.



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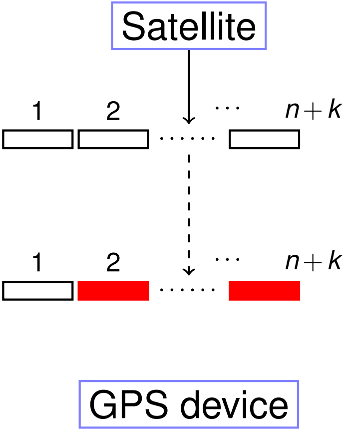

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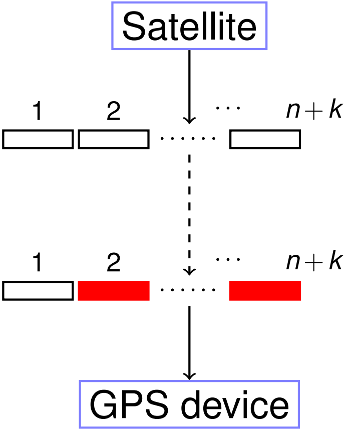

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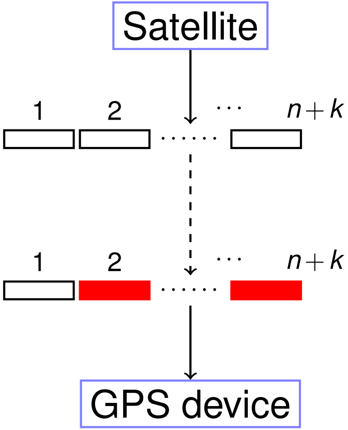

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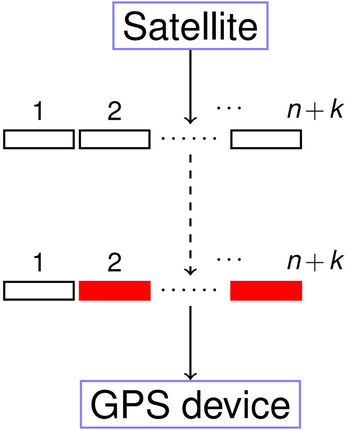

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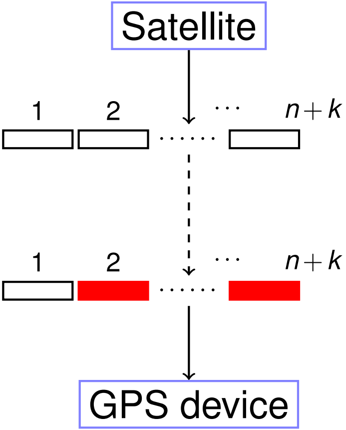

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Send

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

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$a_1 = 2a_0$. $a_0 = 2 \pmod{7}$ $a_1 = 4 \pmod{7}$ $a_2 = 2 \pmod{7}$

$P(x) = 2x^2 + 4x + 2$

$P(1) = 1$, $P(2) = 4$, and $P(3) = 4$

Send
Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

Notice that packets contain "x-values".

Bad reception!

Send: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

Bad reception!

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Recieve: (1,1) (2,4), (6,0)

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Send: $(1, 1), (2, 4), (3, 4), (4, 7), (5, 2), (6, 0)$

Recieve: (1,1) (2,4), (6,0)
 Reconstruct?

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Format: $(i, R(i))$.

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Lagrange or linear equations.

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Channeling Sahai

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Message?

Bad reception!

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Channeling Sahai ...

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Message? $P(1) = 1$,

Bad reception!

Send: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

Recieve: (1,1) (2,4), (6,0)
Reconstruct?

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Questions for Review

You want to encode a secret consisting of 1,4,4.

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You want to send a message consisting of packets 1,4,2,3,0

Questions for Review

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Larger than 144 and prime!

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You want to send a message consisting of packets 1,4,2,3,0

through a noisy channel that loses 3 packets.

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The other constraint: arithmetic system can represent $0, 1, 2, 3, 4$.

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Send n packets b-bit packets, with k errors.

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The other constraint: arithmetic system can represent $0, 1, 2, 3, 4$.

Send n packets b-bit packets, with k errors.
Modulus should be larger than $n + k$ and also larger than 2^b.

Polynomials.

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- ..give Secret Sharing.

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

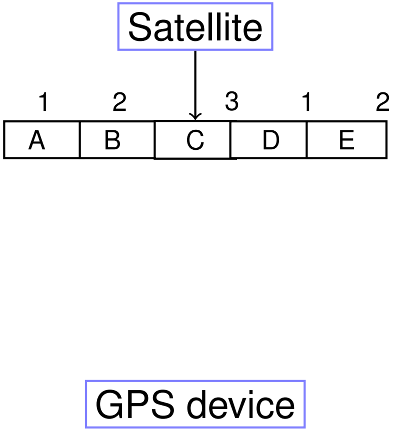
Satellite

3 packet message.

Corrupts 1 packets.

GPS device

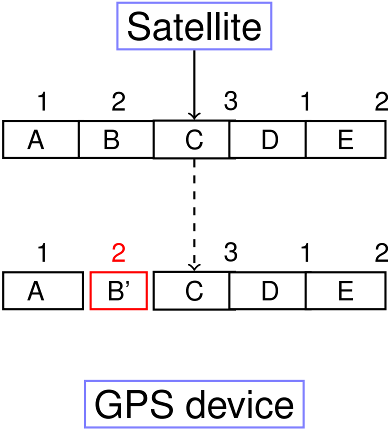
Error Correction



3 packet message. Send 5.

Corrupts 1 packets.

Error Correction



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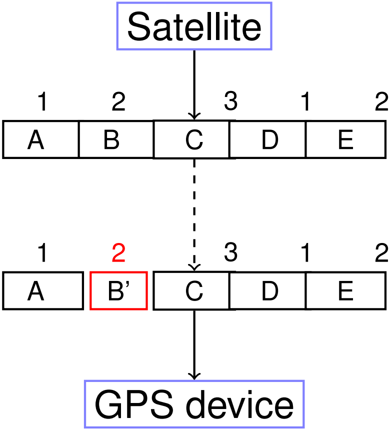

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The Scheme.

Problem: Communicate n packets $m_1, \ldots, m_n$ on noisy channel that corrupts $\leq k$ packets.

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Properties: proof.

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(2) Degree $n-1$ polynomial $Q(x)$ consistent with $n+k$ points.

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$Q(x)$ agrees with $R(i)$, $n+k$ times.

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Total points contained by both: $2n+2k$.

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Total points contained by both: $2n+2k$. P Pigeons.
Total points to choose from : $n+2k$.

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Total points contained by both: $2n+2k$. P Pigeons.
Total points to choose from : $n+2k$. H Holes.

Properties: proof.

$P(x)$: degree $n-1$ polynomial.
Send $P(1),\ldots,P(n+2k)$
Receive $R(1),\ldots,R(n+2k)$
At most k i's where $P(i) \neq R(i)$.

Properties:
(1) $P(i) = R(i)$ for at least $n+k$ points i,
(2) $P(x)$ is unique degree $n-1$ polynomial
that contains $\geq n+k$ received points.

Proof:
(1) Sure. Only k corruptions.
(2) Degree $n-1$ polynomial $Q(x)$ consistent with $n+k$ points.
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Points contained by both : $\geq n$.

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$\implies Q(i) = P(i)$ at n points.

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(Aside: Message in plain text!)

Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

Slow solution.

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For each subset of $n+k$ points

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For each subset of $n+k$ points
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Slow solution.

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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!

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- For any subset of $n+k$ pts,
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  2. and where $Q(x)$ is consistent with $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$
\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod{7} \\
4p_2 + 2p_1 + p_0 &\equiv 1 \pmod{7} \\
2p_2 + 3p_1 + p_0 &\equiv 6 \pmod{7} \\
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Assume point 1 is wrong

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Assume point 1 is wrong and solve..

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Assume point 1 is wrong and solve..no consistent solution!

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Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
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Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong and solve...consistent solution!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!!

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Could be anywhere!!!

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Error!! .... Where???

Could be anywhere!!! ...so try everywhere.

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

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Something like $(n/k)^k$ ...Exponential in k!.

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Something like $(n/k)^k$ ...Exponential in k!.

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
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Oh where, oh where can he be?

Ditty...

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Oh where, Oh where

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With his ears cut short
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Oh where, Oh where

Ditty...

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Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

Oh where, Oh where
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Oh where, oh where can he be

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And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$E(1)(p_{n-1} + \cdots p_0) \equiv R(1)E(1) \pmod{p}$$

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}
$$

Where oh where can my bad packets be?

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&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my bad packets be?

$$
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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4 unknowns (p_0, p_1, p_2 and e),

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Multiply equation i by $(i - 2)$. All equations satisfied!

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
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(p_{n-1} (n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
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...so satisfied, I'm on my way.

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and linear in a_i and coefficients of $E(x)$!

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Solving for $Q(x)$ and $E(x)$...

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```
           -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

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```
                   1 x^2
              -----------------
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```

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        ----------
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```
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         -----------------
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         ----------
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         -----------------
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         ----------
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                ---------------
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```

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```
                   1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
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          -----------------
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                 ---------------
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                           x - 2
                           -----
                               0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x + 1
            -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$

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                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

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         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

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What is $\frac{x-2}{x-2}$?

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                   ---------------
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                             x - 2
                             -----
                                 0
```

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What is $\frac{x-2}{x-2}$? 1

Example: finishing up.

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                 ---------------
                           x + 5
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                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6$.

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

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```
                1 x^2 + 1 x + 1
         -----------------
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         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
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                ---------------
                          x + 5
                          x - 2
                          -----
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$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

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Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:

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Proof:
We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

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$E(x)$ and $E'(x)$ have at most k zeros each.

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Can cross divide at n points.

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$\Longrightarrow \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.
Both degree $\leq n-1$

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Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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for $i \in \{1, \ldots n+2k\}$.

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If $E(i) = 0$, then $Q(i) = 0$.

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If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

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Cross multiplying gives equality in fact for these points.

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Points to polynomials, have to deal with zeros!

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$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1),...P(8)$.

You recieve packets $R(1),...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3+2 = 5$.
(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
(E) The number of coefficients of $P(x)Q(x)$ is 6.

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(E) is false.

(A) $E(x) = (x-1)(x-4)$
(B) The number of coefficents in $E(x)$ is 2.
(C) The number of unknown coefficents in $E(x)$ is 2.
(D) $E(x) = (x-1)(x-2)$
(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

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(D) $E(x) = (x-1)(x-2)$
(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

(A), (C), (E). (F) doesn't type check!

Summary. Error Correction.

Communicate n packets, with k erasures.

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How many packets?

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How many packets? $n+k$
How to encode?

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.

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Of degree?

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover?

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

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How many packets? $n+2k$
Why?

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Why?
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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!