1 Pullout Balls

Suppose you have a bag containing four balls numbered $1,2,3,4$.

(a) You perform the following experiment: pull out a single ball and record its number. What is the expected value of the number that you record?

(b) You repeat the experiment from part (a), except this time you pull out two balls together and record the product of their numbers. What is the expected value of the total that you record?

2 Head Count

Consider a coin with $\mathbb{P}[\text{Heads}] = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is $\mathbb{P}[X = k]$, for some $0 \leq k \leq 20$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X \geq 1]$? Hint: You should be able to do this without a summation.

(d) What is $\mathbb{P}[12 \leq X \leq 14]$?

3 Head Count II

Consider a coin with $\mathbb{P}[\text{Heads}] = 3/4$. Suppose you flip the coin until you see heads for the first time, and define X to be the number of times you flipped the coin.

(a) What is $\mathbb{P}[X = k]$, for some $k \geq 1$?

(b) Name the distribution of X and what its parameters are.

(c) What is the expected number of flips we need before flipping heads for the first time?

(d) What is $\mathbb{P}[X \geq k]$, for some $k \geq 1$?

(e) What is $\mathbb{P}[X \leq k]$, for some $k \geq 1$?

4 The Memoryless Property

Let X be a discrete random variable which takes on values in $\mathbb{Z}_+$ (the positive integers). Suppose that for all $m, n \in \mathbb{N}$, we have $\mathbb{P}[X > m + n \mid X > n] = \mathbb{P}[X > m]$. Prove that X is a geometric distribution. Hint: In order to prove that X is geometric, it suffices to prove that there exists a $p \in [0, 1]$ such that $\mathbb{P}[X > i] = (1 - p)^i$ for all $i > 0$.