CS 70 Discrete Mathematics and Probability Theory
Spring 2022 Koushik Sen and Satish Rao **DIS 0A**

1 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

3 Converse and Contrapositive

Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

4 Equivalences with Quantifiers

Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers.

(a)	$\forall x\ \big((\exists y\ Q(x,y)) \Rightarrow P(x)\big)$	$\forall x\ \exists y\ \big(Q(x,y) \Rightarrow P(x)\big)$
(b)	$\neg\exists x\ \forall y\ \big(P(x,y) \Rightarrow \neg Q(x,y)\big)$	$\forall x\ \big((\exists y\ P(x,y)) \land (\exists y\ Q(x,y))\big)$
(c)	$\forall x\ \exists y\ \big(P(x) \Rightarrow Q(x,y)\big)$	$\forall x\ \big(P(x) \Rightarrow (\exists y\ Q(x,y))\big)$